

THE THEORY OF LIMIT DESIGN

Applied to

MAGNESIUM ALLOY AND ALUMINUM ALLOY STRUCTURES

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INTRODUCTION.

THIS paper is a study of the questions:
(1) Are the simplified equations and design procedure in Limit Design that have been developed for application to structural steel construction, applicable without modification to light metal structures? (2) How do the properties of these metals which are different from those of steel affect the application of the theory? Do they offer any advantages? Any disadvantages? (3) What changes in traditional design procedure are warranted? What changes are imperative?

No claim is made to a complete solution of the design problem, since each particular structure involves a thousand and one details. Only the broad fundamental philosophy is discussed here. The discussion and conclusions are supplemented with a report of a number of experiments on a few extrusions of some of the more important alloys of magnesium and of aluminium.

In view of the radical departure of the Theory of Limit Design from the well-worn and established methods of Applied Elasticity, a brief historical introduction seems justified.

When man first began to build he had no rational methods of proportioning the

members of the structures that he built. He solved his construction problems largely by guesswork and the groping methods of trial and error. It is true that he did possess a certain amount of "engineering judgment," but it was only in regard to relative values, that is, he knew by instinct that of two beams of the same material, whose lengths (spans) were equal, the "stouter" one was the stronger. History⁽¹⁾ tells us that while certain definite relations existed among the dimensions of some structural members, as, for example, pillars, in early (up to the 14th century) building practices, there does not seem to have been any rational basis for these values. Rather they represent practices that had been found satisfactory, and were handed down from generation to generation.

The first to depart from these strictly cut and dried methods was Leonardo da Vinci.⁽¹⁾ He attempted among other things to solve the column problem, which was first successfully solved by Euler two and a half centuries later. The methods da Vinci employed were crude, and whatever conclusions are to be gleaned from the notes he left have not contributed much to the knowledge of the theory of strength.

After da Vinci came Galileo Galilei.^{(1), (2)} While Galilei is most famous for his studies

and experiments in dynamics and astronomy, he did contribute to the science of strength. His was the first rational study of this science, and in his theory he presented with geometric proof a number of propositions concerning the strength of cantilevers, and of beams on two supports. Although most of his conclusions are incorrect because of wrong assumptions, he did arrive at correct results in his comparisons of relative strengths in fracture of similar beams of different dimensions. Among other things he correctly determined the shape of a cantilever beam of constant width, whose resistance to fracture against a load at the free end is uniform along its entire length. His most significant contribution lies in the fact that he was the first to consider the internal forces at the base of the cantilever. He assumed that these forces were uniformly distributed from top to bottom, and that they had a single resultant tension force acting axially at the centre of the beam (he considered beams of rectangular and circular cross sections only). The assumption is wrong, but it was a step forward.

Edmé Mariotte, ^{(1), (2)} who came some time after Galilei, made actual tests on beams to verify or disprove Galilei's propositions. He improved upon the latter's assumption by saying that the internal forces varied linearly from zero at the bottom to a maximum at the top in the cross section of a cantilever. Although he sensed that the fibres on one side of the beam lengthened while those on the other side shortened during bending (again an improvement over Galilei's assumption that the fibres were inextensible), he did not take this into account in considering the distribution of the internal forces. Later in his career Mariotte collaborated with Leibniz, ^{(1), (2)} who believed that the beam fibres were extensible and that "their resistance is in proportion to their extension." This, of course implied that Hooke's Law (1664), which had been formulated some twenty years earlier, applied to the individual fibres

of the beam. This hypothesis came to be called the Mariotte-Leibniz theory.

It is noteworthy that these early investigators of the 16th and 17th centuries were interested only in the capacity resistances of beams. This was natural since they knew little, if anything, about elasticity and the elastic limit stress of materials. When, however, Robert Hooke ^{(1), (2)} gave the world the law that bears his name, the interest was shifted to the elastic behaviour of the material.

Using Hooke's Law as a basis for his work, Jacques Bernoulli ⁽²⁾ made his most important contribution to the bending problem by showing that the curvature of a bent beam varies directly as the strains in its fibres. This was developed later by Euler, who produced a formula equivalent to the present-day equation $1/R = M/EI$. Euler's most famous contribution to the subject is his analytical solution of the column problem, the answer to which P. van Musschenbroek ^{(2), (3)} had anticipated a few years earlier from the results of experiments on columns.

It is highly debatable whether or not Euler consciously used Hooke's Law when he attacked the problem. He was, however, emphatic in pointing out ⁽⁴⁾ that the "moment of stiffness," one of the quantities he used (equivalent to EI in present-day formulas), was a property not limited to elastic beams alone. Euler's problem was peculiar, and he could not escape the fact that he was really determining the capacity load that a column could carry, that is, the load that would induce uncontrolled deformation which for the columns in his day necessarily produced fracture.

It can be said without exaggeration that after Euler very little contribution was made to the subject of strength, insofar as engineering structures are concerned, until the early twentieth century. It is true that nineteenth century history ⁽⁵⁾ is replete with names of the great elasticians—from Charles Coulomb, who gave the first reasonably complete

analysis of the internal forces in a cantilever beam, Thomas Young, who defined Young's modulus of elasticity, Navier who is regarded as the founder of the modern theory of elastic solids, Poisson, who gave the science the famous Poisson's ratio, Clapeyron, who laid the foundation for the development of the three-moment equations for continuous beams, James Clerk-Maxwell, who first proved the special case of Reciprocal Theorem now frequently called Maxwell's Theorem, down to Barré de Saint-Venant, the greatest elastician of them all, whose fundamental assumption of transverse planes remaining planes in bending was revolutionary. In all these things the emphasis has always been on the elastic behaviour of the material, and hence the elastic behaviour of the structure only.

It was not until the dawn of the present century that men began to question the soundness of the concept, which, although it has never been explicitly stated, has nevertheless always been implied, namely, that the exceeding of the elastic limit stress at any one point in a structure, even if the structure is redundant, is tantamount to failure.

All engineers are probably agreed that such is rarely the case and that, with modern structural materials, it is not the case at all. But they are agreed also, that after the elastic limit has been exceeded the formulas of Applied Elasticity no longer hold, and that it becomes necessary to do one of two things: either to stay away from the elastic limit and take comfort in the thought that possibly these formulas apply with sufficient rigour, or, failing to do that in some cases, to adopt certain rules of thumb that years of building practice and experience have justified—rules which have absolutely nothing to do with elasticity, *e.g.*, the stress concentration around a rivet hole is completely ignored and the stress is assumed uniformly distributed around the hole. Unfortunately, neither of these things has been adopted to the complete exclusion of the other.

Instead a sort of compromise has come out of it, and the result is a sorry mixture of theory, based on Applied Elasticity, and some rules which conflict directly with that theory. Understandably, this has led to the statement that there is a conflict between theory and practice. It may be so, but practice cannot be entirely wrong, inasmuch as there are both correct and wrong practices. This does not, on the other hand, imply that the theory is wrong. It would be more proper to say that it is incomplete, since the methods of Applied Elasticity do not take into account the behaviour of structures when they are strained outside the elastic range. In the many steps and processes connected with construction, structural members or parts thereof are frequently strained beyond this range and still retain their usefulness. Thus, the question arises: Is this theory which considers only the elastic behaviour of materials and which completely ignores the ability of these materials, especially of metals, to "give" without breaking, the best possible theory on which to base structural design?

THE THEORY OF LIMIT DESIGN.

The Theory of Limit Design⁽⁵⁾ is a theory of strength which proposes to take into account as fully as possible this "ductility" of metals, and which undertakes to explain the behaviour of structural members or entire structures when they are loaded beyond the proportional limit. It thus enables the designer to take full advantage of the outstanding properties of the materials he works with, and to obtain values very close to the limit or capacity resistances that man first set out to find. This theory, furthermore, rationalises many of the rules of thumb that successful construction has justified. In one sense it is independent of the theory of elasticity and represents an entirely different sense of values—the physical—in contrast to the purely mathematical sense of values represented by this theory of elasticity. In another, larger, sense it is a logical step forward from the concept of elasticity. One

time or another the next step had to be taken, the next question had to be asked: What happens to a structure when at some points it is stressed beyond the elastic limit?

As in most other branches of science, in Limit Design successful practice preceded rational theory. An outstanding example of this is the design of multiple rivet connections. One of the first to philosophise on the suitability or non-suitability of Hooke's Law as a basis for structural design, was Professor N. C. Kist⁽⁶⁾ of the University of Delft, Holland. He posed the question: Does the proportionality of stress and strain as a basis for design in steel construction lead to economical results? After answering this question in the negative, he laid down one of the cardinal concepts of Limit Design: In a redundant structure, it is not necessary to use the equations of elasticity to determine the redundants; it is only necessary to assume values for them, any assumptions at all but preferably the most advantageous ones, provided such assumptions are compatible with the conditions of equilibrium. *In the judicious choice of these assumptions lies the key to the successful application of the Theory of Limit Design.*

Kist thought only in general terms. His published articles do not say whether he made tests in support of his philosophy. As far as can be ascertained the earliest published tests that have any bearing on Limit Design are those performed by H. F. Moore⁽⁷⁾ in 1913. Moore tested simple steel beams only, and the full significance of Kist's principle was not realised. On the other hand, the tests on continuous steel beams made by Maier-Leibnitz⁽⁸⁾ in 1928 proved the correctness of Kist's contention. Furthermore, they provided a guide in the choice of those assumptions for the redundants.

During the time of Kist and the early pioneers there were developed no simple formulas for Limit Design comparable to those for Applied Elasticity. In America at least, it has remained for Professor J. A. Van

den Broek of the University of Michigan to rationalise, systematise, and expand the theory. Indeed, the designation "Limit Design"⁽⁵⁾ is his. His most outstanding contribution is his emphasis on the fact (which many still refuse to admit) that in all construction, antiquated or modern, the elastic limit stress has never been a true criterion of strength. The true and only criterion has always been deformation. Stress comes in only as a secondary item, when the loads or load carrying capacities are computed. This is especially true in redundant structures.

In applying the theory to steel structures Van den Broek offers dictum 3 as follows: "When in a well-designed and especially properly detailed n -fold redundant structure n redundant members are stressed to their elastic limit or up to their critical buckling loads, the deformations involved are of the order of magnitude of elastic deformations until an $(n+1)$ th member reaches its elastic or its critical buckling capacity."⁽⁵⁾

Thus, while Kist and Van den Broek are both right, they differ in approach. Kist was concerned first and last with equilibrium. He knew (and correctly) that if he assumed any values for the redundants and then designed them according to those assumptions, he could determine from statics what the remaining determinate values would be. Van den Broek, on the other hand, wants to know more about what actually happens to the structure as the loads on it are increased until deformations become excessive. In knowing this behaviour he is in a better position than Kist was in making the most advantageous assumptions for the redundants.

APPLICATION TO STEEL STRUCTURES.

Modern structural metals may be divided into two groups: ductile and brittle. Strictly speaking there is none among these metals that is completely brittle, like glass. It is always possible to produce some permanent set, be it ever so small, in every metal. Probably the one which approaches glass

closest in brittleness is cast iron. On the other extreme there is at least one metal which exhibits to a considerable extent perfectly ductile behaviour. This is mild or hot rolled steel. The other structural metals, notably the light metals, fall between these two extremes.

The early portion of the stress-strain curve of structural steel, in both tension and com-

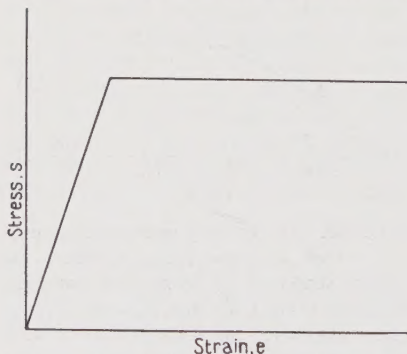


Fig. a.

pression, is represented in Fig. (a). The sharp knee is probably a simplification, this region being one of instability.⁽⁹⁾ The significant fact is that the horizontal portion extends to a value of the strain, e , sometimes up to twenty times the elastic limit strain, here taken for all practical purposes as equal to the proportional limit strain.

This remarkable property of steel enables structural members made of it to behave under load in a simple manner which may be summed up thus: In the elastic range the load and the corresponding deformation vary linearly; after the elastic range there is a transition into a sharply defined condition in which the deformation increases without any appreciable increase in the load. This condition is so clear cut that it is a logical choice for a limit. Expressed differently, this means that the load-deformation diagram for any member of a steel structure is qualitatively the same as the stress-strain curve, with or without the knee, but with the same con-

siderable "flat" portion. What actually happens in the transition is not of primary concern here. The important fact is that a member made of structural steel is capable of developing its particular limit performance. The application of the Theory of Limit Design to steel structures is based on the implied assumption that some particular member, having reached its limiting condition, *continues* to function in this limiting condition and to contribute a *constant* resistance to increasing load until as many other members as can "catch up" finally do catch up (Van den Broek's dictum 3).

In a beam this limiting condition is associated with a limit moment which is peculiar to the particular form of the cross section of the beam and which depends on the yield point of the steel of which it is made. A section which has reached such a condition is sometimes referred to as a "yield hinge." This implies that, if such a section is part of a redundant beam, it will act much as a hinge would, with the difference that instead of transmitting a zero bending moment it will transmit a constant bending moment. The limiting condition for the entire beam is reached when as many sections as can "equalise" their bending moments are developing simultaneously the same resisting moment.

The peculiar behaviour of a beam section just described leads to an extremely simple solution of the redundant beam problem. Kist's principle says: Make the most advantageous assumption for the redundants. Obviously the most advantageous assumption for any one section of a beam is its own limit resisting moment. Van den Broek's dictum 3, in effect, says: Assume as many equalised bending moments as there are redundants plus one, for it is only then that the deformations of the beam cease to be of the order of magnitude of elastic deformations.

In any one span of any beam there cannot possibly be more than two redundants. This

follows from the fact that the conditions of equilibrium can yield only two independent equations—if it is assumed that there are no axial forces and no twisting moments—and that there can be at most only four restraints: two shearing forces and two bending moments. In other words, n in Van den Broek's dictum 3 cannot be greater than 2. In any continuous beam n equals 2 for an interior span and 1 for an exterior span, if for the exterior span the exterior support is considered a simple one.

The following rule is now offered as a guide in the application of Limit Design to continuous beams: *For any one span of any continuous beam, construct the bending moment diagram on the basis that this span is considered as a simple beam. On this diagram superimpose on the same side of the reference line the bending moment diagram (a straight line) due to the extra restraint, or restraints. Adjust this line until two equal moment intercepts for an exterior span, or three equal moment intercepts for an interior span, are obtained which are the lowest possible values for any number of combinations, but which are the highest possible values in that particular combination. These particular sections are the ones in which the equalised bending moments are to be assumed. The remaining quantities in the system can then be determined from the equations of statics.*

This rule is a logical consequence of the fact that the variation of the bending moments, as the loads are increased from zero, is continuous; therefore, the distribution of these bending moments must pass through a combination of minimum values before a combination of maximum values is reached. The usual load combinations encountered in practice should present no difficulties. However, a system may arise in which it is not obvious where to assume the equalised moments. The following simple, though somewhat far-fetched example, will serve to illustrate this rule.

Figure (b) represents the exterior span of a continuous beam carrying the loads

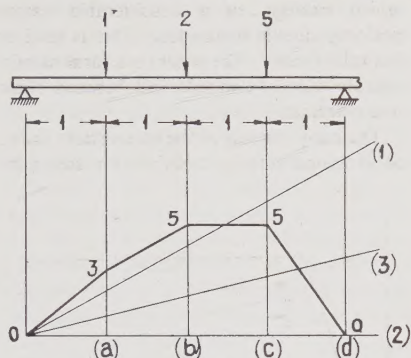


Fig. b.

indicated. The bending moment diagram is constructed with the beam considered as simply supported. The moment ordinates, from left to right, are 0, 3, 5, 5, and 0 units. Assuming the left end of the continuous beam to be simply supported, the bending moment diagram, due to the added restraining moment at the right support, is represented by an inclined straight line starting from zero at the left end. There are three possible positions for this line, each of which yields two equal moment intercepts. They are marked (1), (2) and (3). Position (1) yields two equal moments, one at (a) and one at (b), but these are not the greatest values in this particular combination since the moment at (d) is greater than either. Position (2) gives two equal intercepts, one at (b) and one at (c), but these values are higher than those at (b) and at (d) given by position (3). Therefore, (b) and (d) are the sections in which equal bending moments are assumed, and the design proceeds from there.

In the application of the theory to framed structures, in which compression and tension members are involved, it must be pointed out that the stipulation of a limiting condition for the entire structure is contingent upon the ability of the compression members to behave like long columns subject to centrally applied

loads (pin connections). The load-deformation curves of long steel columns resemble the stress-strain curve,⁽¹⁰⁾ i.e., immediately after a long column buckles it continues to carry practically 100 per cent. of its buckling load, and carries this load for a considerable extent of its deformation in the direction of the load (see Fig. 16 of reference 10). The longer the column is, the longer is the extent to which it does this. This "ductile" behaviour of a long column thus permits a buckled member to behave in the manner described earlier. This is the basic philosophy that has governed the design of transmission towers, a unique field of structural engineering wherein Limit Design has found its freest expression and in which its conscious application has been crowned with glowing success.⁽¹¹⁾

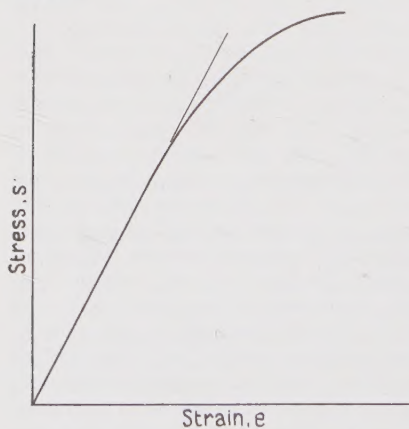


Fig. c.

If the columns are not long columns and the loads are not centrally applied, as is usually the case in practice, then the exact determination of the limiting conditions for such members will have to await further developments in column research. In the meantime, assumptions for the load carrying capacities of these short non-pin connected columns must continue to be based on empirical formulas and intelligent guesswork.

As for the connections in framed structures, if there is any fixity, it can at best be estimated only from the stiffness of the members connected. The extent to which the assumptions for the redundants are actually realised in the limit depends on the accuracy of such an estimate. Exploring further the implications of the theory of Limit Design, the conclusion is that a structural member is capable of ductile behaviour only to the extent to which the connections will allow such behaviour. Hence, the guiding principle in designing connections is: *Connections should be designed "not only so as to develop as nearly as possible the full strength of the members, but so as to develop the full ductility of the members as well."*⁽¹²⁾

APPLICATION TO MAGNESIUM ALLOY AND ALUMINIUM ALLOY STRUCTURES.

The light metal alloys are neither completely brittle nor completely ductile. Like steel they are capable of almost perfectly elastic behaviour, but when stressed past the elastic limit the relation between load and corresponding deformation is entirely different from that in structural steel. Qualitatively the stress-strain curve in tension as well as in compression, with a few exceptions, is generally as shown in Fig. (c).

This characteristic immediately brings out one significant fact, that excessive deformation does not begin at any one point in the stress-strain diagram. What is true of the stress-strain curve necessarily holds also for any one load-deformation curve of members (except columns) made of these metals. The conclusion is inevitable that some value of deformation must be decided upon arbitrarily as a criterion of permissible deformation in determining the range of usefulness of structures made of the alloys of magnesium and of aluminium.

The limiting condition for any light metal structural member must be defined, taking into consideration either the total deformation it may suffer during service, or the total

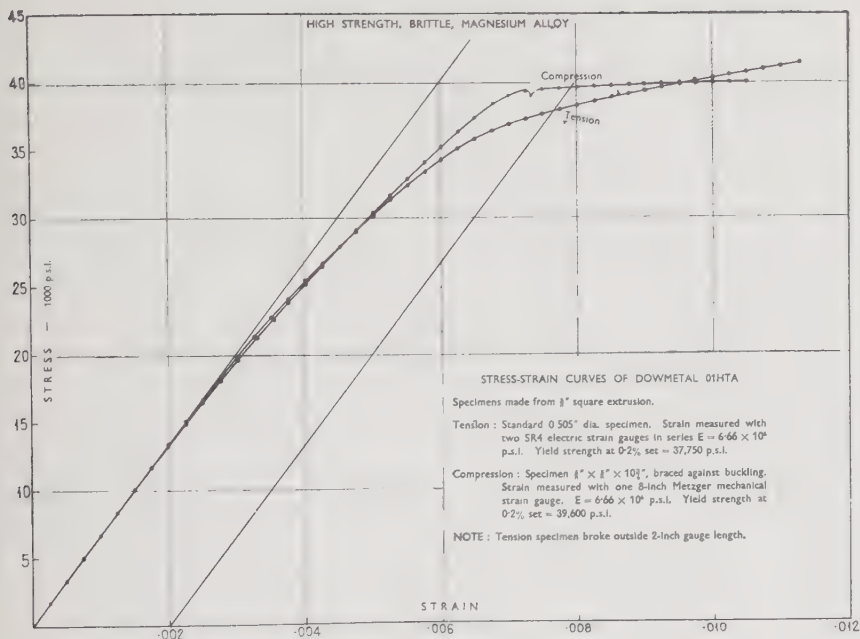


Fig. 1.

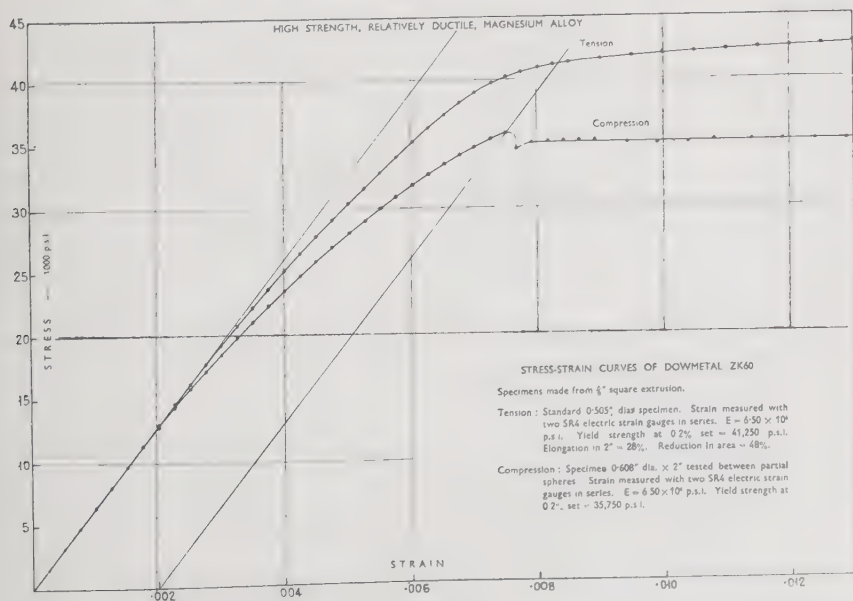


Fig. 2.

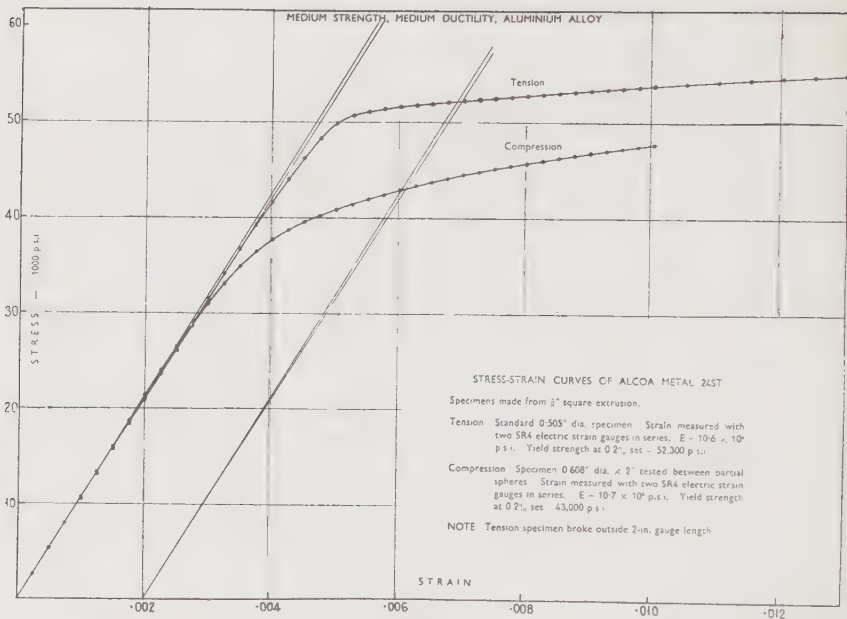


Fig. 3.

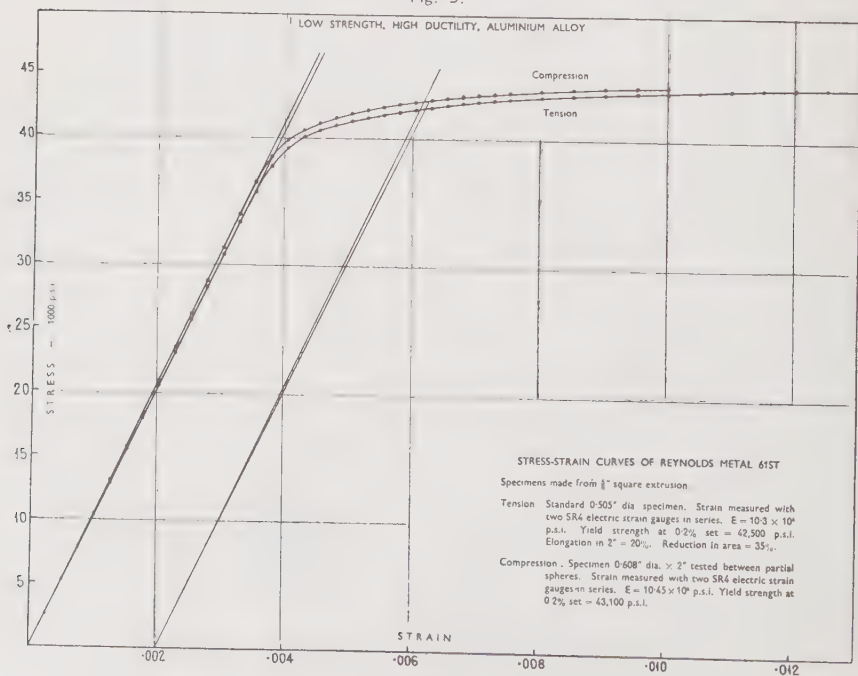


Fig. 4.

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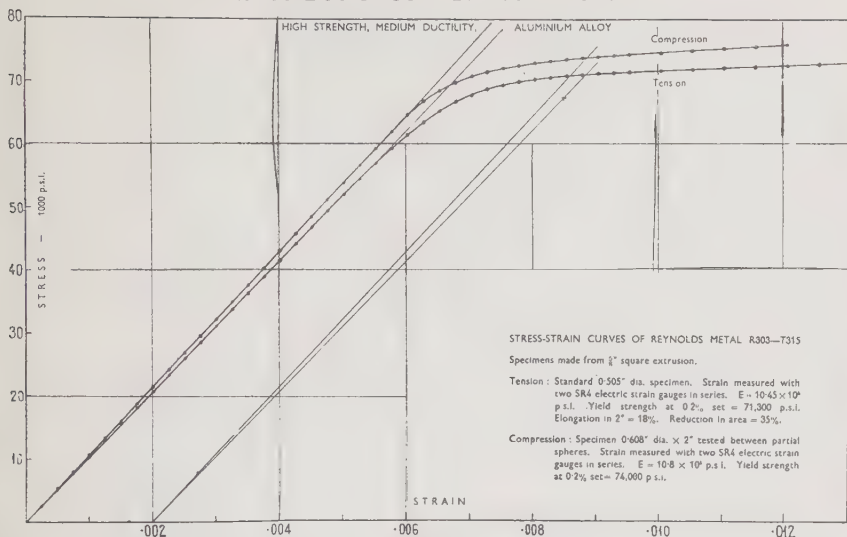


Fig. 5.

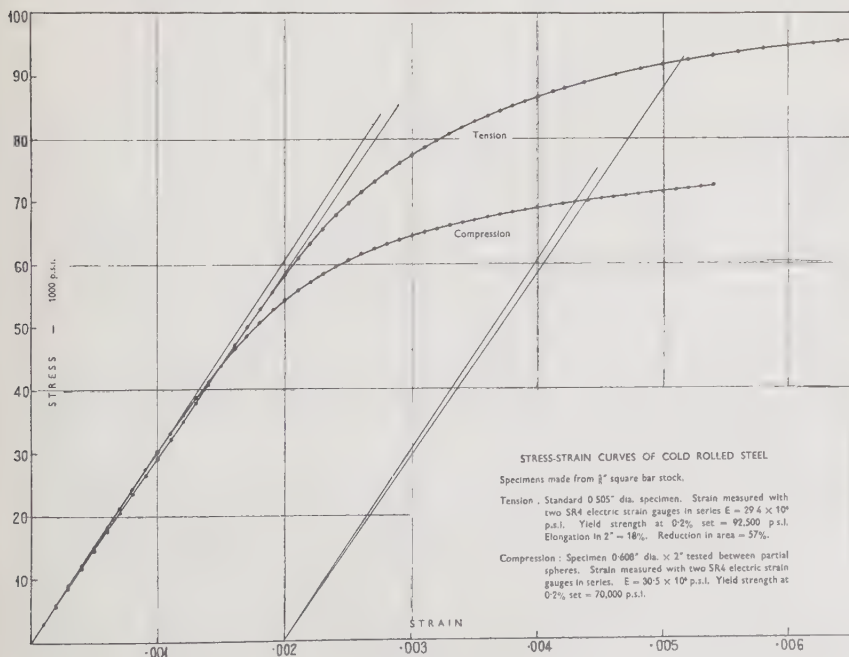


Fig. 6.

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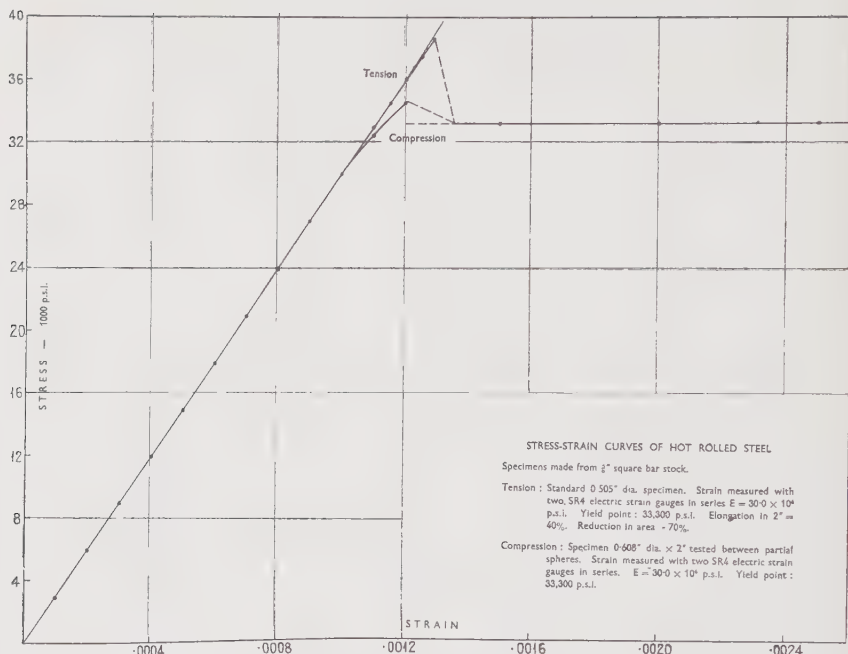


Fig. 7.

permanent deformation it may be allowed to retain. Any way in which this condition may be defined, the arbitrary approach is inescapable. Thus the first glaring difference between the properties of the light metals and those of steel is recognised. In structural steel members there is defined beyond any doubt a condition in which deformations become uncontrolled *without the threat of fracture being necessarily involved*. The load or resisting moment corresponding to such a condition may be thought of as a property inherent in the particular section or member. This represents a service which the member does not necessarily have to perform, but which it is potentially capable of performing if called upon to do so.

On the other hand, in the alloys of magnesium and of aluminium there is no such well-defined point. Deformations in members made of such metals become excessive only

when they exceed some value that will have to be agreed upon in advance, and in no sense do these deformations become "uncontrolled" except at, or very near, fracture. Therefore, the limit loads, or limit moments, likewise have to be arbitrarily defined. The fact has to be faced that the very nature of the stress-strain characteristics of these metals precludes any exact determination of limit loads, mathematical or experimental. In the laboratory an individual isolated member could be so tested as to develop its ultimate performance, but such performance would be difficult if not impossible to evaluate in terms of actual service conditions when such a member is an integral part of a larger structure.

The presence or absence of such a well-defined condition of yielding, however, is a matter of only secondary importance in design. The considerations on which light

must be shed really are:—What assumptions may be made for the strength of structural members of magnesium alloys and aluminium alloys. For any one such assumption how much more or less is the margin of safety in terms of the probable potential load carrying capacity. To throw light on these matters the following propositions are offered. Tests to substantiate these propositions will be discussed later.

First, the fact that, insofar as primary failures are concerned, the use of the elastic limit stress as a criterion of limiting conditions is no longer indicated in redundant steel structures, means that it is even less indicated in redundant light metal structures. Second, the very nature of the load-deformation characteristics of the alloys of

tinuing ability of the structural members to develop increasing resistances.

In evaluating the strength of structures it is necessary to have a suitable yardstick. In view of the fact that no hard and fast limits for allowable deformations are at hand, the most logical thing to do is to compare the deformations in a redundant system with the deformations in a statically determinate system of similar dimensions under loads that are identically applied. Such a choice is natural since the present-day concept of elastic limit stresses and working stresses is a carry over from the days of statically determinate structures. The proposed yardstick is a suitable statically determinate system subject to a certain combination of loads. The reason for this is that if the

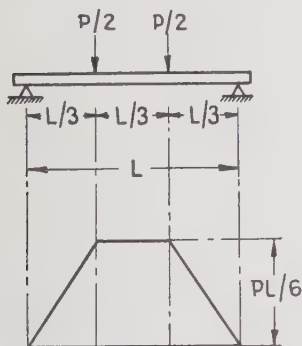


Fig. d.

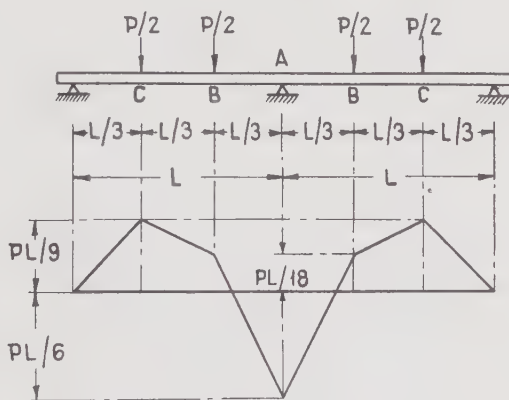


Fig. e.

magnesium and of aluminium allows the application of Kist's principle to an even greater advantage than does steel. This second proposition simply means that while the presence of the well-defined yield condition may be desirable for the sake of having a clear cut limiting state on which to base working values (loads, and so on), its absence actually constitutes an advantage in that relatively higher loads can be carried by the redundant structure because of the con-

tinuing ability of the structural members to develop increasing resistances. In evaluating the strength of structures it is necessary to have a suitable yardstick. In view of the fact that no hard and fast limits for allowable deformations are at hand, the most logical thing to do is to compare the deformations in a redundant system with the deformations in a statically determinate system of similar dimensions under loads that are identically applied. Such a choice is natural since the present-day concept of elastic limit stresses and working stresses is a carry over from the days of statically determinate structures. The proposed yardstick is a suitable statically determinate system subject to a certain combination of loads. The reason for this is that if the

EXPERIMENTS: SERIES A AND SERIES B.

There are many possible combinations of simple beams and of the corresponding redundant beams which are all easily set up in the laboratory. The combination which makes the strongest appeal is that shown in Figs. (d) and (e). Fig. (d) represents a simple beam on two supports carrying two loads at its third points. Fig. (e) represents a beam on three supports at the same level whose two spans are equal to the span of the simple

beam. This second beam carries two loads in each span also applied at the third points.

This set-up appeals for two reasons. First, complete fixity, which in the classroom is just an academic abstraction, is beautifully and conveniently realised in the symmetry of the redundant beam, *i.e.*, one half of this beam may be considered as a beam which is completely fixed at one end and simply supported at the other end. Second, within the elastic range the maximum bending moment in the

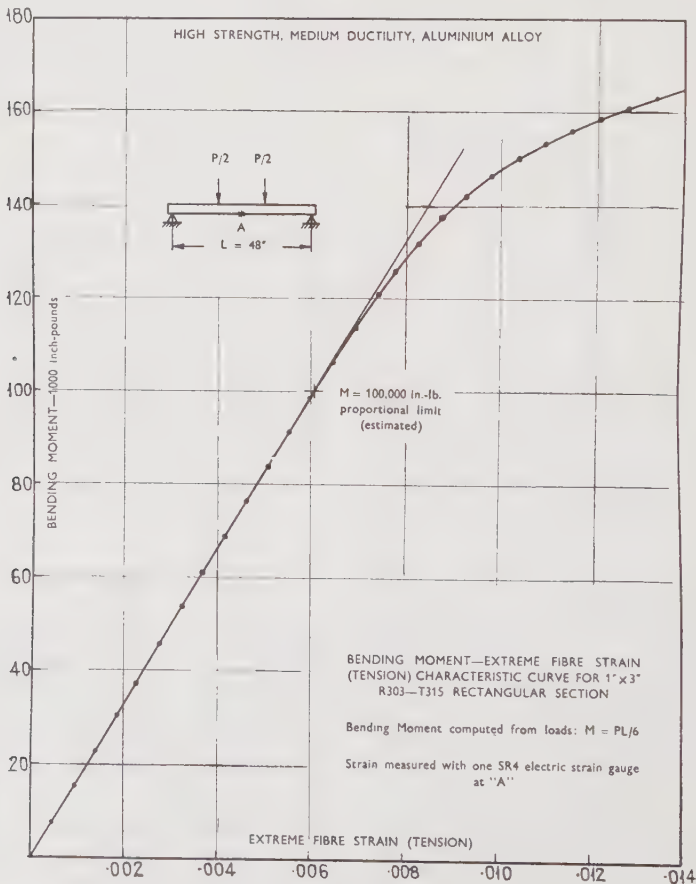


Fig. 8.

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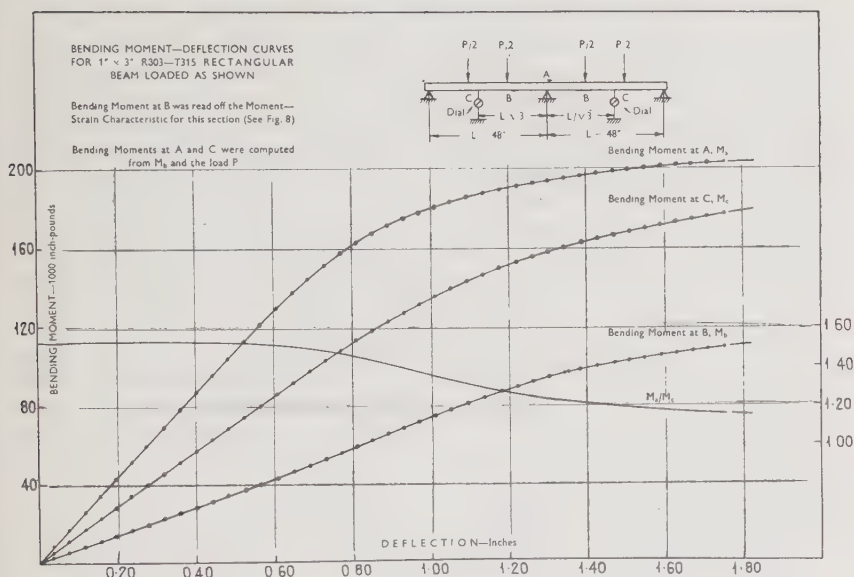


Fig. 9.

redundant beam is the same as the maximum bending moment in the simple beam, both equal to $PL/6$. Thus, according to the elastic limit stress criterion of strength, both redundant and statically determinate beams should be of the same strength. The test records manifest at a glance the error involved in this criterion.

SERIES A (Figs. 8 to 17).

The object is threefold. First, to demonstrate the property of the light metals, described earlier, which makes the load-deformation diagram of a light metal structural element (in this case a beam) similar to the stress-strain curve. Second, to prove that because of this property the redundant-plus-one sections in a beam never equalise moments within reasonable limits of deformation even for low initial ratios of the bending moments, that is, ratios of these moments in the elastic range as given by the equations of Applied Elasticity. Third, to

verify experimentally Van den Broek's dictum 3.

The following extrusions were tested: 1 in. x 3 in. Reynolds aluminium alloy R303-T315 rectangular section, $L=48$ in. (see Figs. (d) and (e)); 1 in. x 3 in. Reynolds aluminium alloy 61ST rectangular section, $L=48$ in.; $\frac{5}{8}$ in. x $\frac{5}{8}$ in. Dowmetal magnesium alloy ZK60 square section, $L=12.5$ in. The rectangular bars were tested in the 200,000 lb. Rhie universal testing machine of the Materials Testing Laboratory of the University of Michigan. The square bars were tested in the 72,000 lb. Amsler universal hydraulic testing machine. The deflections were measured with Federal dial gauges. The surface strains were measured with SR4 electric strain gauges (See Plate I).

The following readings were taken:— the total load P_1 for the simple beam, the deflection at the centre, and the extreme fibre strain in tension at the centre also; the total load $2P_2$ for the redundant beam, the deflections

at two symmetrical points whose distance from the centre support was $L/\sqrt{3}$ (the deflection is maximum at these two points within the elastic range), and the extreme fibre strains in tension at the sections B.

The following curves were plotted:—, a characteristic curve showing the variation of the resisting moment with the extreme fibre strain in the simple beam (Figs. 8, 11 and 15); a series of curves showing the variation of

the bending moments at the three sections A, B, and C as well as the variation of the ratio M_a/M_o with the deflection in the redundant beam (Figs. 9, 12 and 16); a load-deflection curve for the simple beam and one for the redundant beam (Figs. 10, 13 and 17). The deflections for the redundant beam are the averages of the two dial readings. The moment (M_b) at B was read off the moment-fibre strain characteristic curve for mean

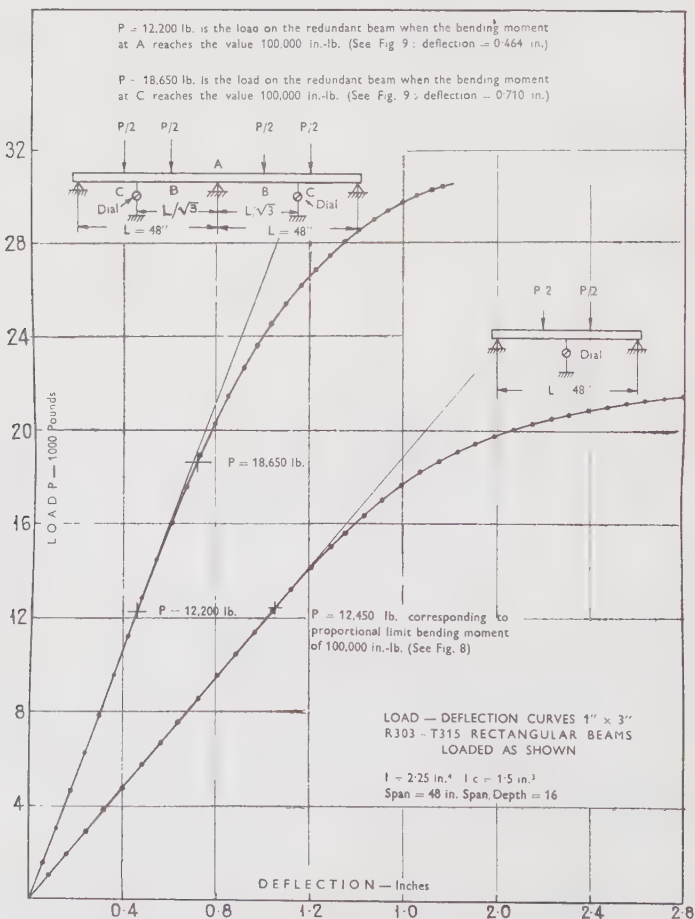


Fig. 10.

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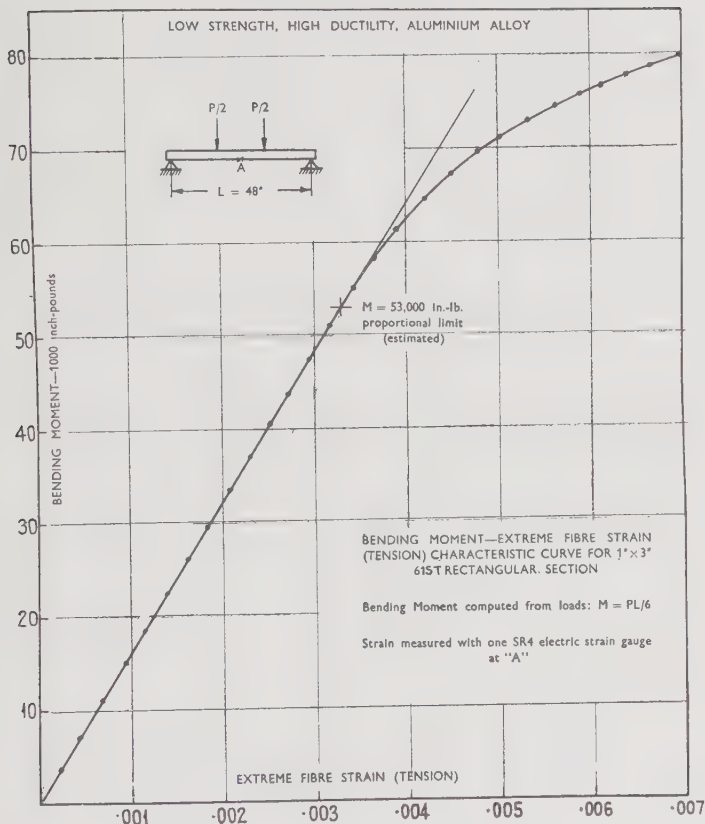


Fig. 11.

values of the strain as given by the two SR4 electric gauges at the sections B, while the moments M_a and M_o were computed from M_b , the known load P , and the equations of statics.

INTERPRETATION OF THE RESULTS.

First: Figs. 8, 11, 15, 10, 13 and 17 all show that the load-deformation (or the moment-fibre strain) curves for these alloys are similar to the stress-strain curves, which are given in Figs. 5, 4 and 2. All these curves, with the exception of the compression stress-strain curve of Dowmetal ZK60 (Fig. 2) have

no "knees." This detail, as also that of Dowmetal 01HTA (Fig. 1), has significance only in explaining the shift of the neutral axis of the beam while bending past the elastic limit. This is not of primary concern here.

Second: From Figs. 9, 12 and 16 it is clear that in spite of the low initial ratio of M_a to M_o (which analytically should be $PL/6 \div PL/9 = 1.5$, but which experimentally came out to be 1.52, 1.53 and 1.41, respectively, for the three tests), M_o does not quite catch up with M_a ; and although the curve for M_a/M_o does show a tendency to be asymptotic to the horizontal line through

1.00, such an equalisation of moments obviously would occur only at prohibitive deflections. The necessity for an arbitrary definition of limiting conditions should be perfectly evident from these curves. Fig. 14 shows a comparison of the M_n/M_0 curves of the R303-T315 and 61ST aluminium alloy sections. They tend to the same value. The fact that the 61ST curve dips sooner to a lower level follows from the lower strength of this alloy. Compare the stress-strain curves Figs. 5 and 4.

Third: Figs. 10, 13 and 17 each has marked on it three points—one for the simple beam and two for the redundant beam. The point on the simple beam curve corresponds to the load when the centre section first develops the proportional limit resisting moment, which is determined from the moment-fibre strain characteristic. The lower point on the redundant beam curve corresponds to the reaching of the proportional limit bending moment at section A (redundant section, $n=1$), while the upper point corresponds to

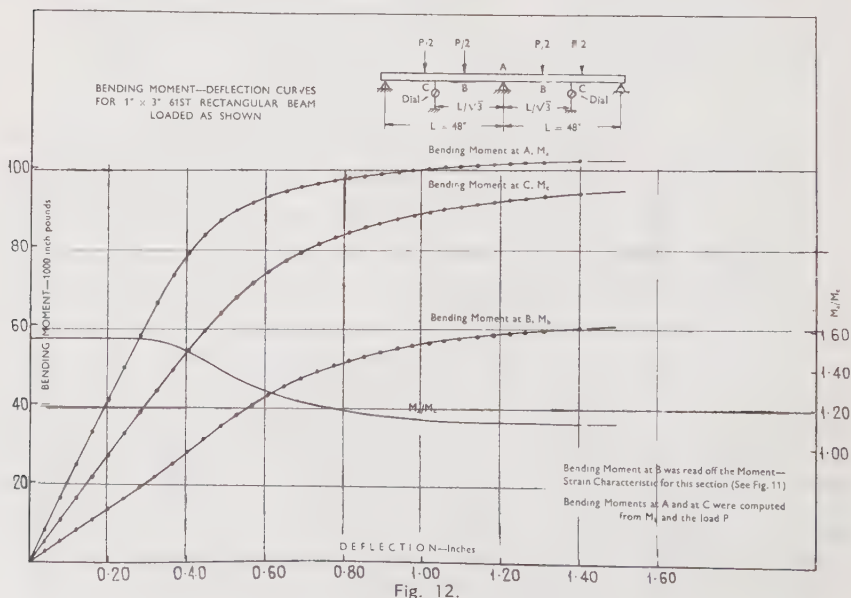
the reaching of this bending moment at the next critical section, $C(n+1=2)$.

It is noteworthy that even under the loads corresponding to this second point the deformations in the redundant systems are still of the order of magnitude of the elastic deformations, *i.e.*, the deviation from the original straight line is still very small, and there is definitely no tendency to "run away." What is more striking is the fact that under these "limit" loads the deformations in the redundant systems are still much smaller than the proportional limit deflections in the simple beams!

The implication is that Van den Broek's dictum 3 applies with even greater force to light metal structures than it does to structures of mild steel.

SERIES B (Figs. 18 to 23).

The object is two-fold. First, to show that the elastic limit stress is an extremely inadequate criterion of the strength of a redundant structure and this is especially so



THE THEORY OF LIMIT DESIGN

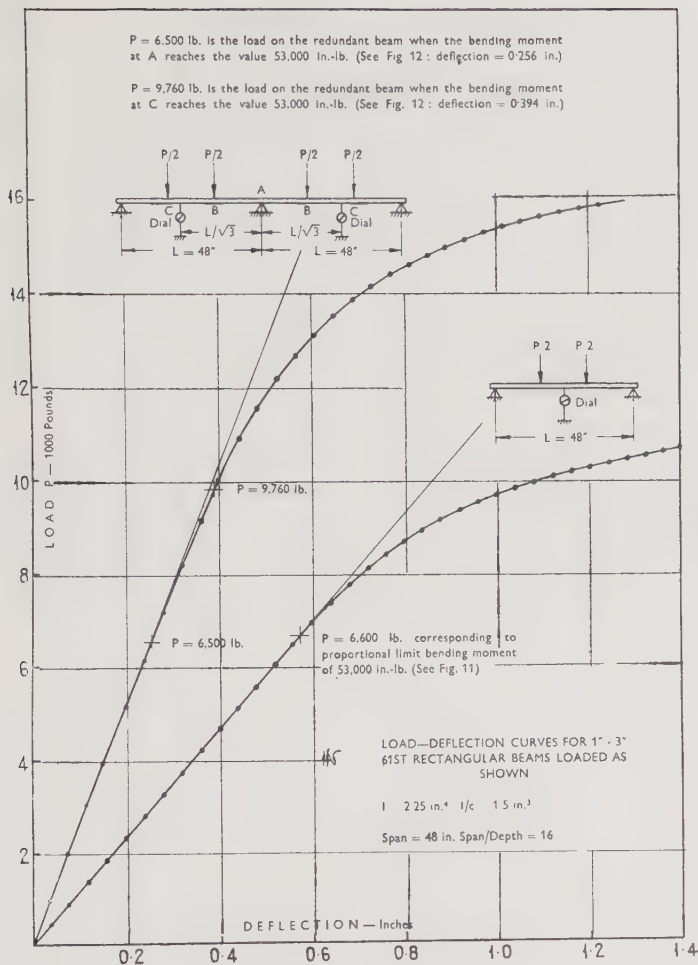


Fig. 13.

in the case of structures made of the alloys of magnesium and of aluminium. Second, to demonstrate the truth of the statement made earlier that the absence of the well-defined condition of yielding in the light metals actually constitutes an advantage, in that relatively much higher loads can be carried by redundant structures made of these metals

(as compared to similar statically determinate structures) than can be carried by redundant structures of mild steel (as compared to similar statically determinate structures).

The following extrusions were tested:—
 Dowmetal magnesium alloy ZK60, Dowmetal magnesium alloy 01HTA, Alcoa aluminium alloy 24ST, Reynolds aluminium alloy 61ST,

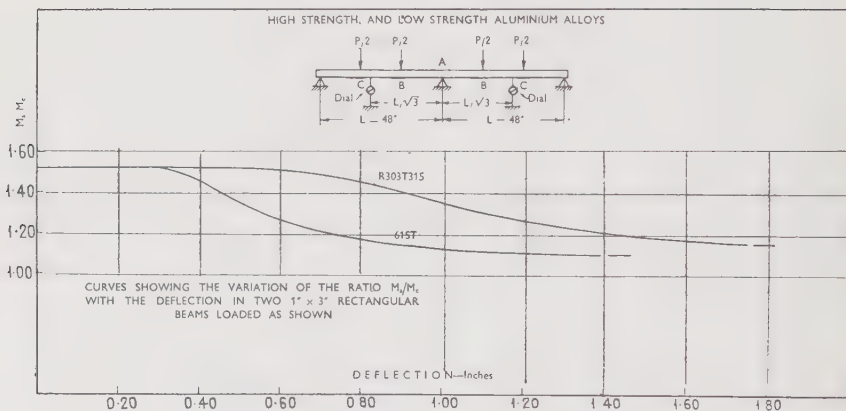


Fig. 14.

Reynolds aluminium alloy R303-T315, and bars (from stock) of cold rolled steel and hot rolled steel. All bars were $\frac{5}{8}$ in. square and all spans were equal, $L = 12.5$ in., giving a span-to-depth ratio of 20 (see Figs. (d) and (e)). All tests were run in the Amsler

universal hydraulic testing machine with the 7,200 lb. scale. Deflections were measured with Federal dial gauges.

The following readings were taken:—total load P_1 for the simple beams, deflection at the centre; total load $2P_2$ for the redundant

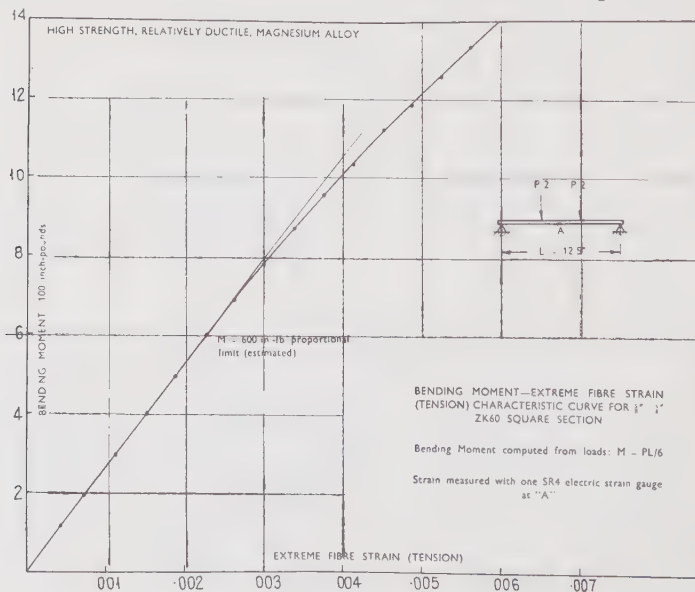


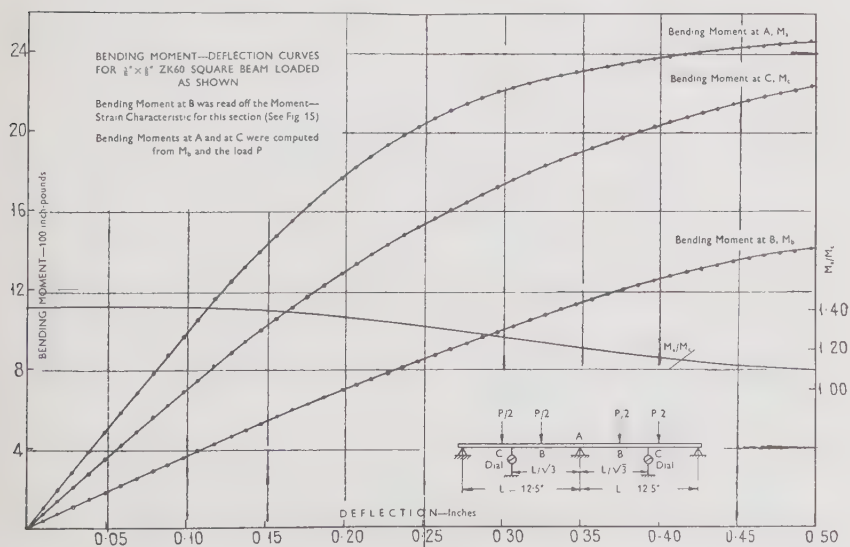
Fig. 15.

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Plate I.

A 1 in. x 3 in. rectangular extrusion being tested as a beam on three supports. Span, $L=48$ in.



beams, deflections at the two symmetrical points $L/\sqrt{3}$ distant from the middle support.

The following curves were plotted:— a load-deflection curve for the simple beam; a load-deflection curve for the redundant beam, the deflections used being the averages of the two dial readings; and a curve showing the variation of the ratio of the load carrying

capacities of the two beams for the same deflection as this deflection increases. These curves are shown three to a sheet on Figs. 17, 18, 19, 20, 21, 22 and 23, respectively, for the seven alloys.

INTERPRETATION OF THE RESULTS.

An examination of any one of Figs. 18, 19, 20, 21, 22 and 23 will reveal three points

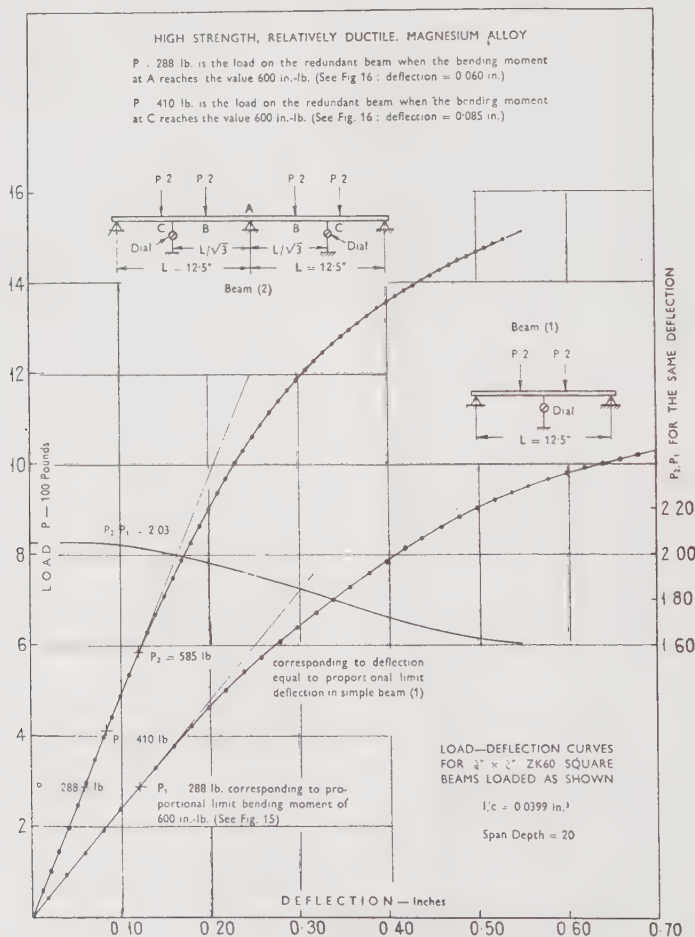


Fig. 17.

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marked out, one on each of the three curves described above. The point on the simple beam curve corresponds to the load carried when the proportional limit in the centre section of the beam is first reached. This point was estimated from the curve in these six figures. In the case of the Dowmetal ZK60 extrusion (Fig. 17), which was run as part of Series A also, this proportional limit

was estimated from the moment-strain characteristic curve (see discussion of Series A tests). The point on the redundant beam curve corresponds to a load which causes a deflection equal to the proportional limit deflection in the simple beam. The point on the P_2/P_1 curve shows the ratio of these two loads. This is in a sense a direct comparison of the load carrying capacities of

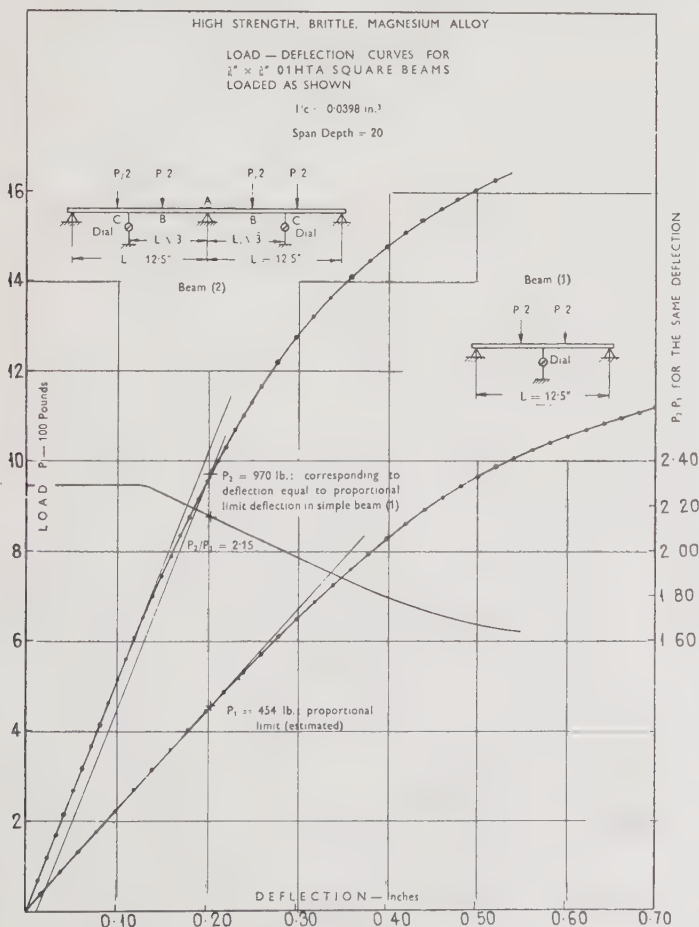


Fig. 18.

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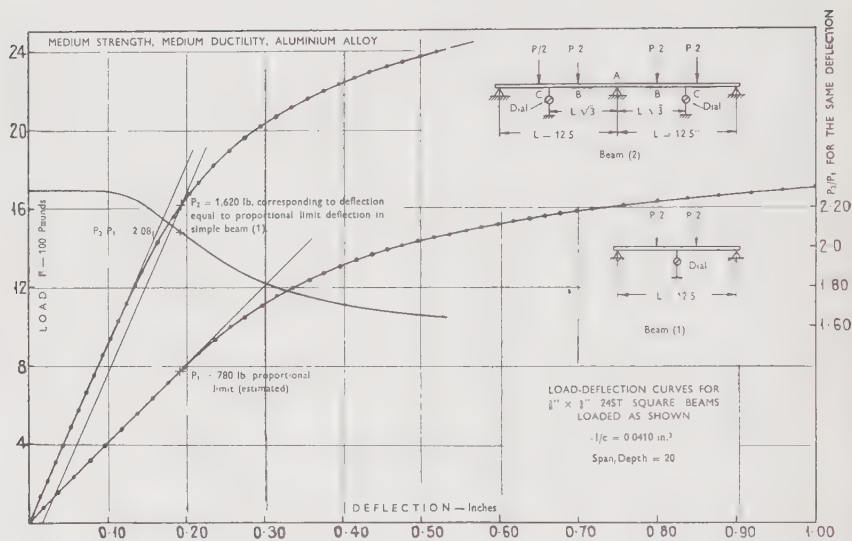


Fig. 19.

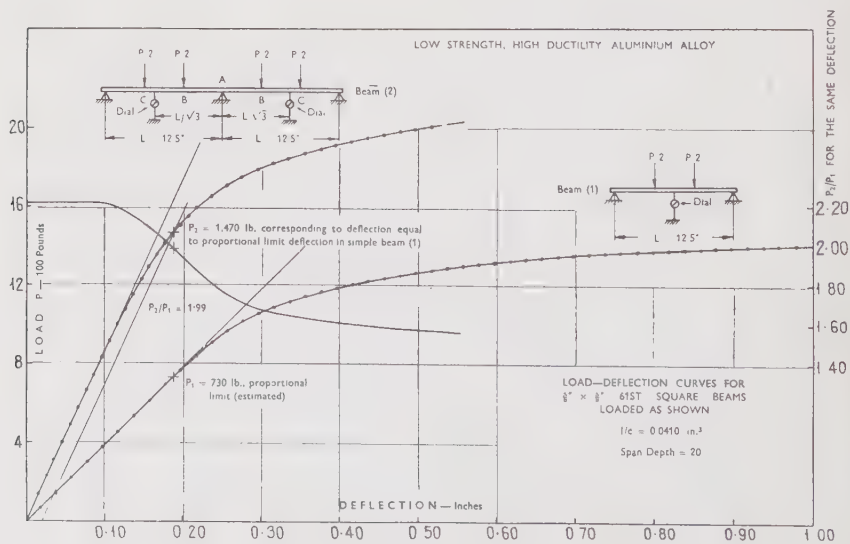


Fig. 20

THE THEORY OF LIMIT DESIGN

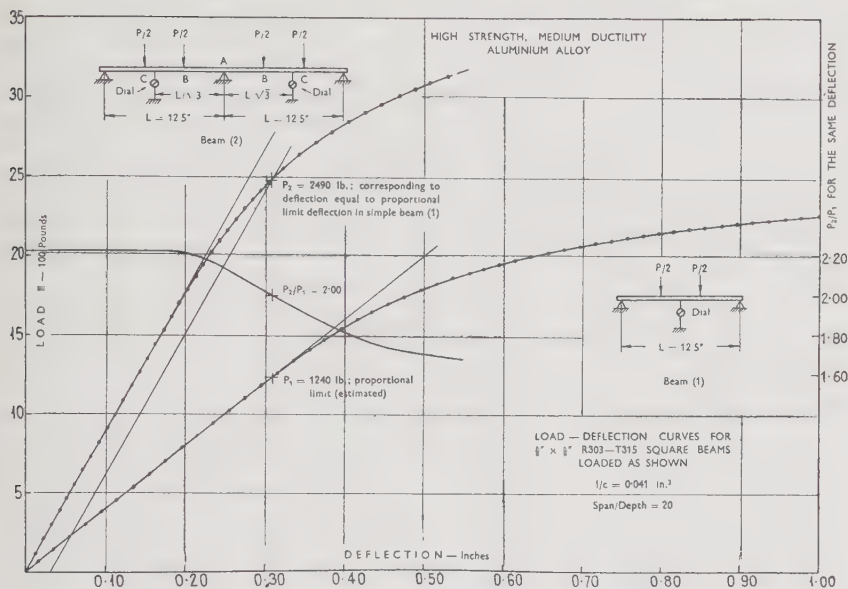


Fig. 21.

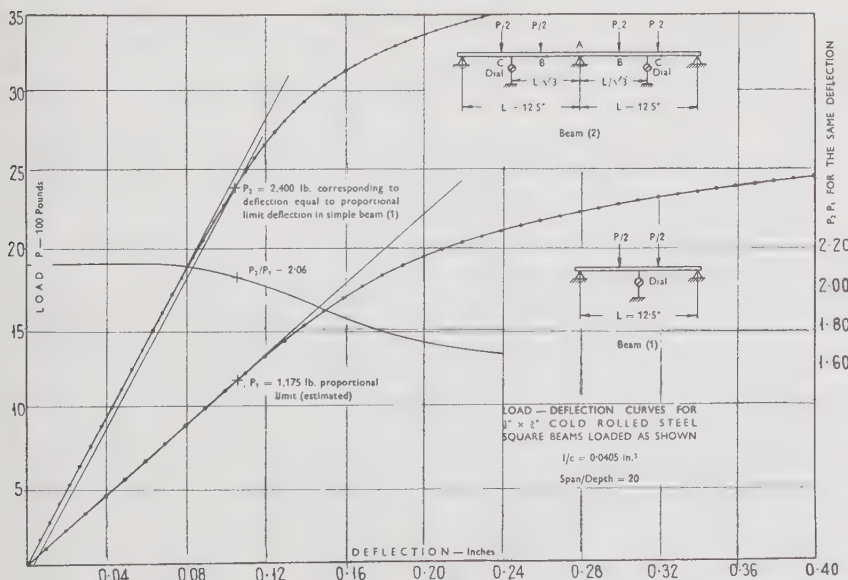


Fig. 22.

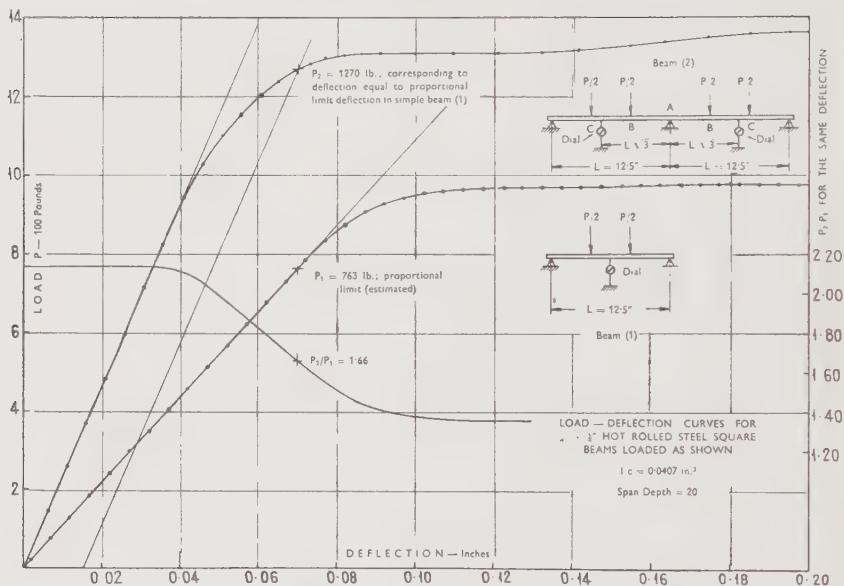


Fig. 23.

the two systems for the same maximum deformations. The following conclusions may be drawn from these curves.

First: If the load carrying capacity of the redundant beam is based on the elastic or proportional limit stress, its strength is the same as that of the simple beam. This means that the potentiality of the redundant system is exploited only to a pitifully small extent. On the other hand, if the load carrying capacity of this redundant beam is based on a maximum deflection equal to the proportional limit deflection of the simple beam (the yardstick), it is evident from the curves that even under such loads the resulting deformations in the redundant systems are still of the order of magnitude of elastic deformations, that is, the deviation from the original straight line is still very small. In no case is this deviation more than 0.03 in. (Fig. 21, for R303-T315), or $3/12.5 = 0.24$ per cent. of the span. The ratio of this load to

that based on the proportional or elastic limit stress ranges from approximately 1.66 for the hot rolled steel (Fig. 23) to approximately 2.15 for the Dowmetal magnesium alloy 01HTA (Fig. 18). Hence, the elastic or proportional limit stress is a poor criterion of strength in redundant structures, especially so in such structures made of the alloys of magnesium and of aluminium.

Second: The presence in hot rolled steel of a definite yield point leads to a definite condition of yield in the structure, be it simple or redundant. Fig. 23 shows this clearly for beams. The limit load for the simple beam is approximately 970 lb., while that for the redundant beam is definitely 1,310 lb. Thus the redundant system may be said to be 35 per cent. stronger than the simple system, and there cannot possibly be any ambiguity in such a statement, these two conditions being well defined. The P_2/P_1 curve becomes horizontal at 1.35. (Analytically this ratio

is 1.33.) In contrast with this none of the light metals, including the cold rolled steel, has such a definite yield condition.

In order to compare the capacities of the two systems, simple and redundant, what better yardstick is there than the proportional limit deflection in the simple beam? Therefore, the load P_2 in the redundant system, corresponding to this proportional limit deflection in the simple beam, is compared with P_1 in the simple system which causes this deflection. This ratio averages around 2 for the five light metals and the cold rolled steel. It is only 1.66 for the hot rolled steel.

These values are only approximate, inasmuch as no attempt was made to correct for the lack of rigidity of the testing machine when the deflections were plotted. But such a correction would only tend to increase the steepness of the redundant beam curves, thus showing to still better advantage the superiority of the light metals, *i.e.*, the value of P_2 for the same deflection would be much higher. Such details were dispensed with on the grounds that they would distract attention from the main issues of this dissertation. Thus the statement that the load carrying capacity of redundant beams made of the light metals is relatively higher, because of the continuing ability of the beam sections to contribute increasing resisting moments, is completely proven.

DESIGN OF BEAMS.

From the foregoing discussion and experiments it is implied that the simplified procedure in designing steel structures, more specifically the rule offered as a guide in the application of Limit Design for the design of continuous steel beams, may be applied to light metal construction without any modification other than that some value of the stress be decided upon—low enough so as to avoid excessive deformations but high enough so as to make use of the material as efficiently as possible—to take the place of the yield point stress in structural steel. It is proposed here to use for the stress criterion

the yield strength at 0.2 per cent. offset in tension, in tension for no reason other than that this is a comparatively easier test than compression, and 0.2 per cent. offset because this is the most commonly mentioned deviation in specifications and guaranteed minimum properties.

The assumption of equalisation of moments is not quite justified as in the case of steel in which moments can, within limits, actually equalise; and certainly one may not always assume equalised moments for *all* load combinations. But for the sake of illustration this assumption is made here. The limitations of such a procedure will be discussed under Evaluation of the Theory of Limit Design.

In making an assumption of the bending moment which a section may offer in resistance to loading, there are four methods available to guide the designer. The first and most logical, is by direct experiment. That is, the load-deflection curve of some simple beam (which could easily be standardised) could be obtained, and some value of the bending moment could be decided upon as a useful property of that particular section. Such a bending moment, for example, could be that corresponding to the load when the deviation from the original straight line in the load-deflection curve is a certain percentage of the span, say 0.2 per cent.

The second method is based on the assumption of an equalised distribution of stresses from one side of the beam to the neutral axis and an identical distribution of stresses of opposite sign on the other side—the so-called rectangular distribution of stresses.^{(13), (14)} This assumption can be practically satisfied only in the case of structural steel, and the more closely the stress-strain curve of the material approaches that of steel the greater the justification in using this method.

The third method is one proposed by F. P. Cozzone⁽¹⁵⁾ and further elaborated on by the engineering department of the Curtiss-Wright

Corporation.⁽¹⁴⁾ It consists in an assumed trapezoidal distribution of stresses (an approximation of the characteristic shape of the stress-strain curves of the light metals), based on an implied arbitrarily chosen value of maximum stress. The method is simple only for rectangular sections; it becomes tedious when applied to other sections, such as I-sections.

The fourth method is based on the assumption that the usual Applied Elasticity formula, $M=sl/c$, applies past the elastic limit, thereby utilising the fictitious stress as something analogous to the so-called modulus of rupture, only in this case there is no rupture involved. This last method is clearly the most practical inasmuch as the usual handbooks prepared by the different manufacturers already contain the necessary I/c values.

It is proposed here to use this last method in making the assumption for the resisting

moment that may be assigned to a beam section. Having made this assumption the design follows the same procedure as in the case of steel beams. The following experiments were designed to determine more or less how much margin of safety such a procedure leaves in both the redundant and the simple beams, and to point out the possible lines of attack in a research programme that could very well be undertaken by the large manufacturing concerns.

EXPERIMENTS: SERIES C (Figs. 24 to 30).

This series of experiments, like the first two series, involves the test of a simple beam (the yardstick) and of a redundant beam. For the sake of convenience, the load combination was made to be much simpler than in the preceding experiments. The two beams and the corresponding bending moment diagrams (in the elastic range for the redundant beam) are shown in Figs. (f) and (g).

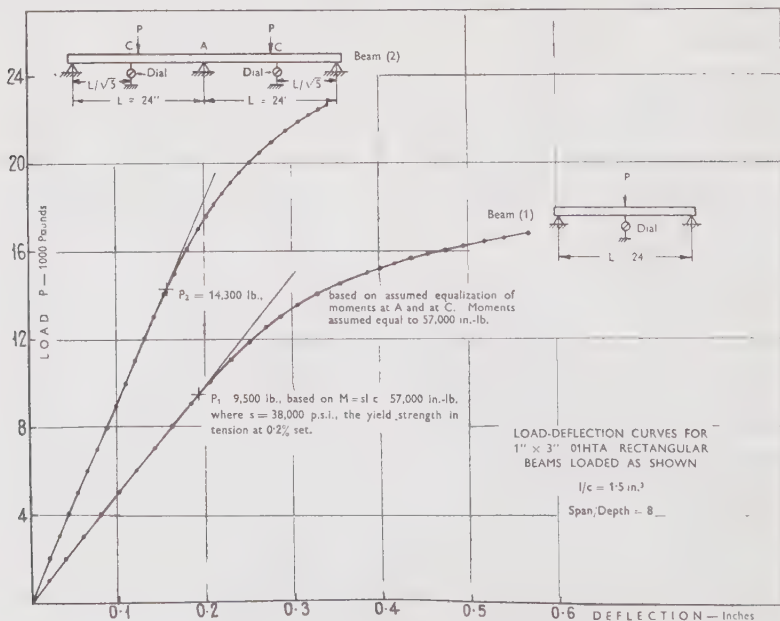


Fig. 24.

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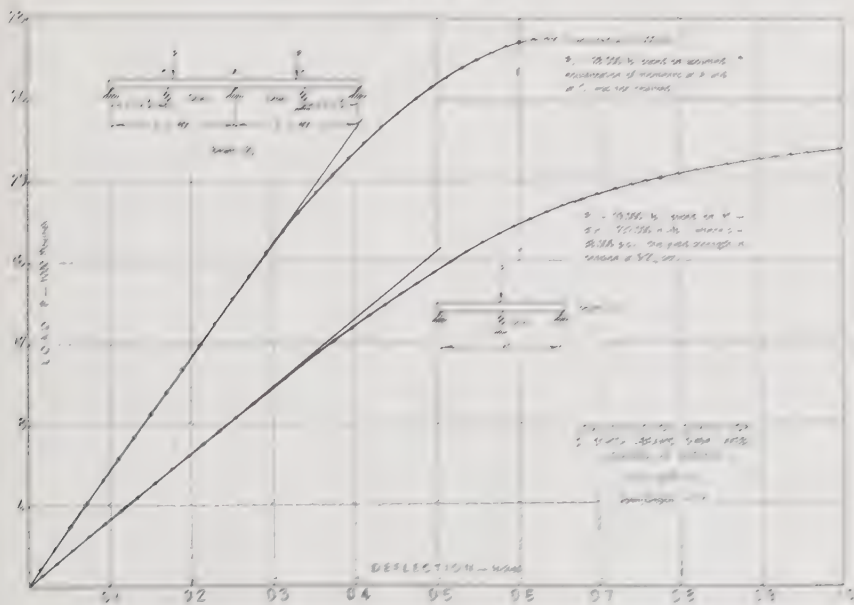
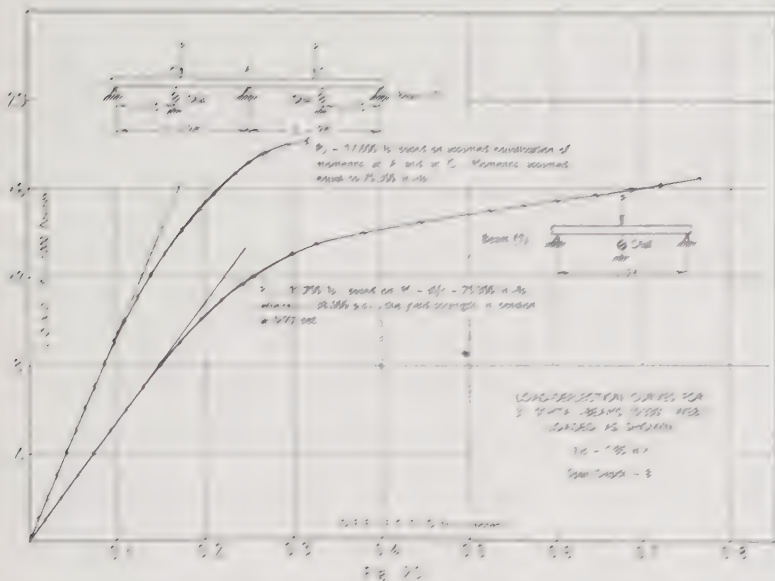


Fig 25

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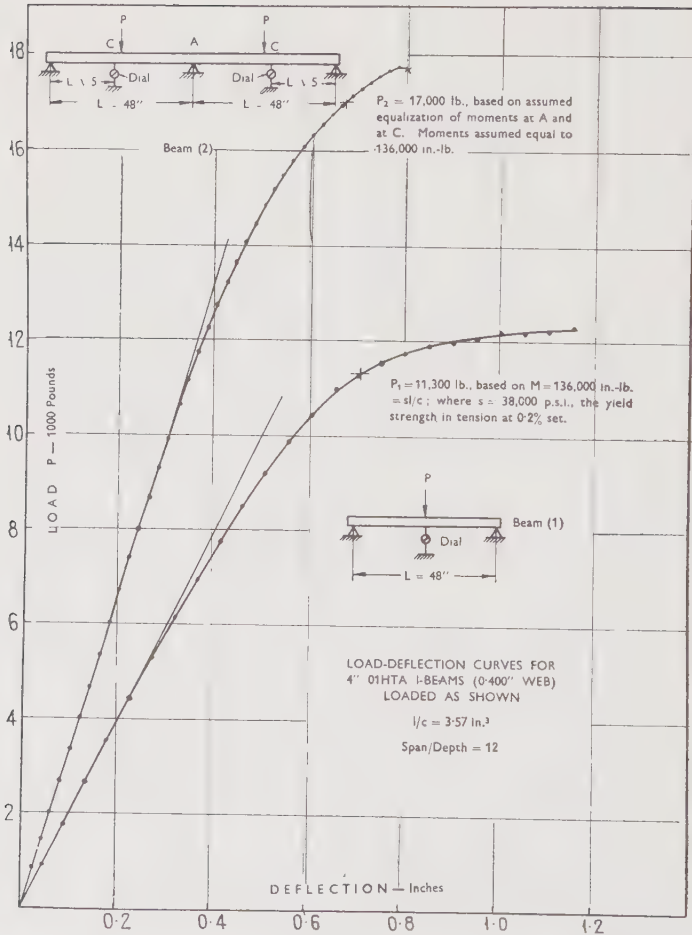


Fig. 27.

The accompanying table shows the extrusions tested, the spans of the beams, and the span-to-depth ratios.

Test No.	Section.	Alloy	Span, L	Span/Depth
1	1" x 3" rectangle	01HTA magnesium	24"	8
2	3" I (0.330" web)	01HTA "	24	8
3	5" I (0.494" web)	01HTA "	49	9.8
4	4" I (0.400" web)	01HTA "	48	12
5	3" I (0.330" web)	01HTA "	48	16
6	1" x 3" rectangle	61ST aluminium	48	16
7	3" I (0.153" web)	61ST "	48	16

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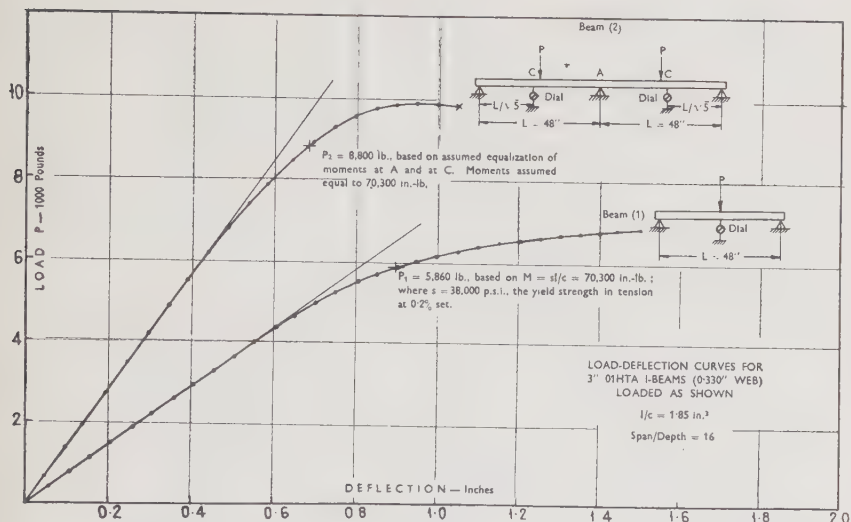


Fig. 28.

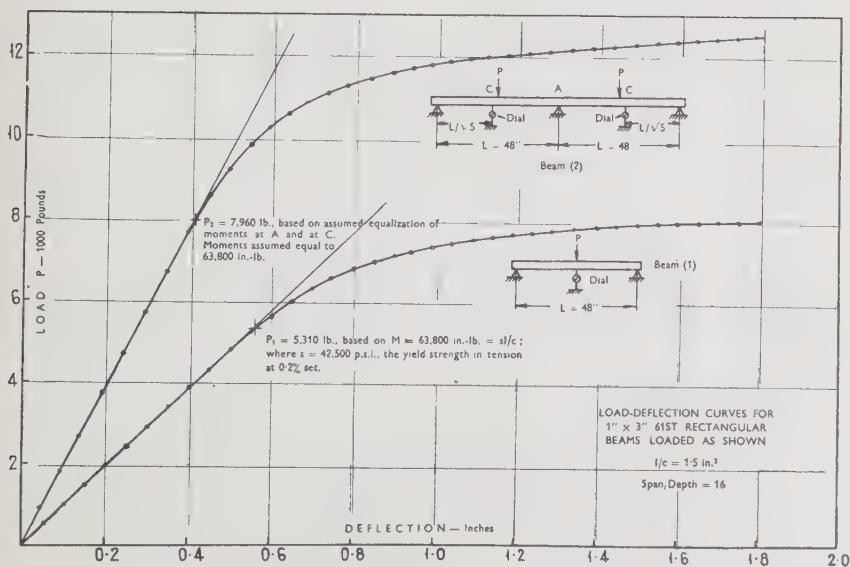


Fig. 29.

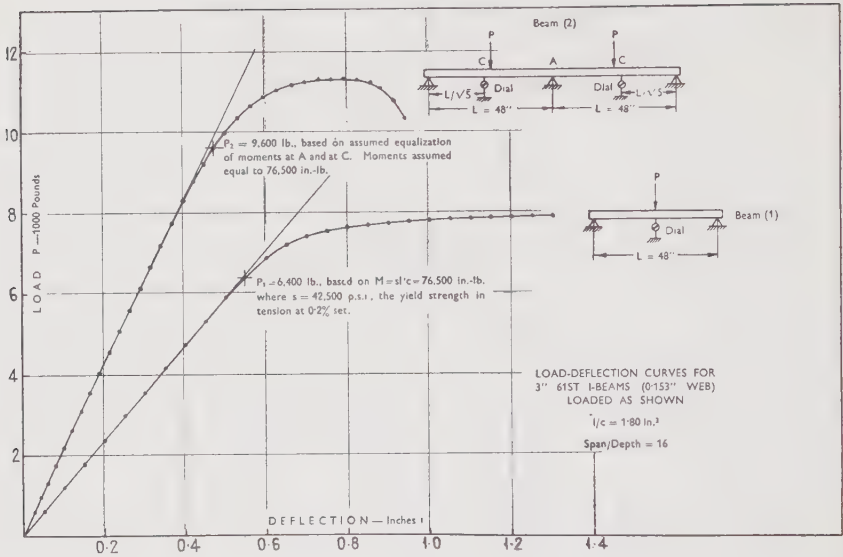


Fig. 30.

The beams with 24-inch spans were run in the Amsler hydraulic machine. All the rest were run in the 200,000 lb. Rhieie machine with the exception of the 1 in. $\times$ 3 in. 61ST rectangular simple beam, which was run in a 10,000 lb. Tinius Olsen hand-bending machine. All deflections were measured with Federal dial gauges. The deflections in the redundant beams were taken at two

symmetrical points whose distance from the outer supports was $L/\sqrt{5}$. These are the points of maximum deflection in the elastic range. No correction was made for the shift of these points after the passing of the elastic limit, nor for the lack of rigidity of the testing machine.

The following readings were taken:— the load P_1 for the simple beam, the deflection

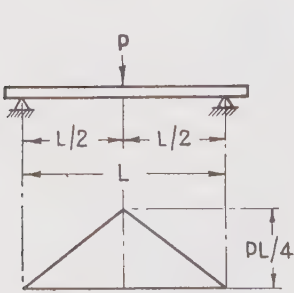


Fig. f.

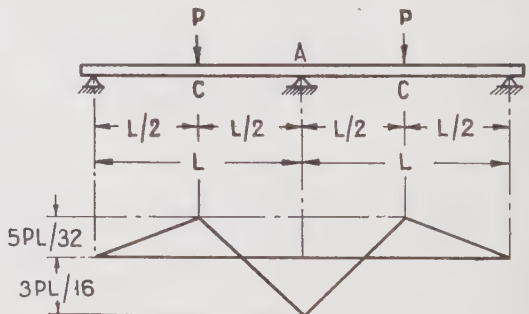


Fig. g.

at the centre; the total load $2P_2$ for the redundant beam, and the deflection at the above-mentioned points.

The following two curves were plotted:— a load-deflection curve for the simple beam, and a load-deflection curve for the redundant beam, the deflections plotted being the averages of the readings of the dial gauges. These curves are presented in Figs. 24 to 30 in the order of listing in the above table.

INTERPRETATION OF THE RESULTS.

In tests 1, 4, 5, 6 and 7 it is seen from the curves (Figs. 24, 27, 28, 29 and 30, respectively) that the loads P_2 on the redundant beams, which were based on the assumed equalisation of bending moments at sections A and C (Fig. (g)), produced deflections no greater than those in the simple beams under the loads P_1 based on the moment $M = sl/c$, where for s the yield strengths at 0.2 per cent. offset in tension were used. (See the stress-strain curves given in Figs. 2 and 4.) In test 2 the resulting deformation of the redundant system (Fig. 25) was greater than the corresponding deformation in the simple system, while in test 3 (Fig. 26) the expected value of the load based on equalisation of moments was not realised. This beam failed laterally and fractured locally under one of the loading blocks—no doubt because of a shortcoming in the test set-up.

For the particular loading used the conclusion may be drawn that, in the interests of safety, the method of assumed equalisation of moments should not be used for span-to-depth ratios less than 12 for the standard I-extrusions of Dowmetal 01HTA magnesium alloy. This raises the question of the safety factor.

The underlying philosophy of Limit Design, which is concerned with the determination of true or arbitrarily chosen limiting conditions for a structure, implies that the factor of safety should be applied not on the elastic limit stress to obtain a working stress, but rather on the limiting loads, thereby

obtaining working loads. In design this indicates that the safety factor should take the form of an overload factor by which the given loads (determining which, incidentally, is itself also a vital aspect of design) are multiplied to obtain maximum loads. The structure should then be so designed that under these maximum loads it be brought as close to the limiting conditions as possible.

Returning to the tests under discussion, if a safety factor of, say, 2 is used, it is seen that in all seven systems, simple as well as redundant, the loads are well within the proportional limit. Thus it may be expected that under such loads these systems will function without any permanent deformations. Further examination of Figs. 25, 26, 27 and 28, all for Dowmetal 01HTA magnesium alloy I-sections, will reveal a little cross at the end of each redundant beam curve. This cross indicates fracture, invariably local fracture caused by concentrated pressure from a loading block or from the middle support. No such fracture occurred in any of the aluminium alloy sections. Hence, it should also be stated here that, in the case of 01HTA magnesium alloy extrusions, probably much larger safety factors should be applied in view of the comparative brittleness of this particular alloy. This will be discussed further.

EXPERIMENT: SERIES D (Fig. 31).

This last experiment was prompted by the startling discovery that the existing I-sections in aluminium and magnesium handbooks are practically identical copies of the sections which the steel industry has gradually developed and found acceptable. The differences, which are extremely minor, are changes in the fillet radii. This much change has been recognised as necessary by the designers. It makes one wonder whether any thought has ever been paid to the fact that failures of members with thin sections invariably follow from local buckling, or crippling, in compression of these sections. Whether

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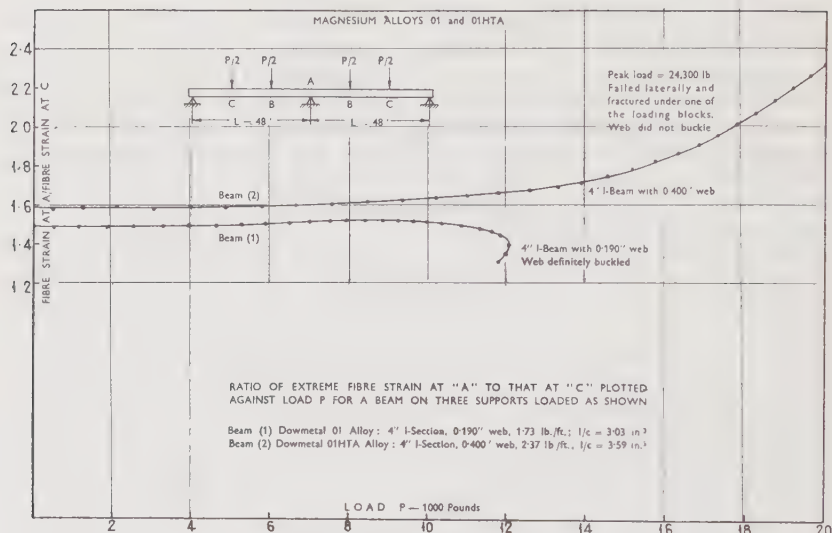


Fig. 31.

such local buckling is elastic or inelastic, it is well known that the modulus of elasticity, or some modification of it, enters into the picture. Therefore, it is imperative that provision be made for the lower moduli of the light metals, in that the thinnest sections should accordingly be made heavier than they are in steel.

Two 4 in. I-sections with different web thicknesses were tested as identical beams on three supports. The set-up was similar to that used in Series A and Series B (Figs. (e) and 31). One beam was of Dowmetal 01 magnesium alloy with a 0.190 in. web. The other was of Dowmetal 01HTA magnesium alloy with a 0.400 in. web. Three SR4 strain gauges were installed on the tension side of the beam at sections A and C.

Readings for the total load $2P$ and for the strains at the three sections were taken, and the following curve was plotted for each beam:— the variation with the load P of the ratio of the extreme fibre strain at A to that at C, using for the strain at section C the mean of the strain readings at the two

symmetrically located sections C. These two curves are shown in Fig. 31.

Strictly speaking, it is not correct to compare the two alloys since the yield strengths at 0.2 per cent. offset in compression are different. An unpublished report of the Dow Chemical Company, compiled in September 1943, gives values for this stress of 23,000 p.s.i. for the 01 alloy and 36,000 p.s.i. for the 01HTA alloy (not guaranteed minimum properties). The tension yield strengths are about the same. Notwithstanding this difference, the ratio of the two yield strengths in compression is so much smaller than the ratio of the two web thicknesses that it is still possible to compare the two beams, at least qualitatively.

From Fig. 31 it can be seen that, under a load of about 8,500 lb., beam (1) with the thin web began to develop local failure. The web began to buckle in section A where the bending moment and the shear were highest. This is evident from the fact that the curve starts to fall. The ratio of the strain at A to that at C is an index of beam behaviour.

That is, for pure beam action, as for example in a solid rectangular section, the strain at A should continually grow at an increasing rate, so that the ratio of this strain to the strain at section C would continually increase also. In beam (2) with the heavy web this is substantially what happened—the curve keeps rising. In beam (1) the strain at A reached a peak under a load of approximately 12,000 lb., which was also the peak load. Under this load the web over the centre support was definitely kinked, and this remained even after unloading. On the other hand, beam (2) developed a peak load of 24,300 lb. without buckling of the web. It failed laterally and fractured locally at one flange under a loading block—again a flaw in the test set-up.

The difference between the two performances was so marked as to justify the conclusion that the section with the heavier web is the more efficient one, *e.g.*, from the standpoint of the strength-to-weight ratio: $24,300/2.37 = 10,200$, while $12,000/1.73 = 6,920$ only. [See Fig. 31. 2.37 is the weight in pounds per foot of beam (2), while 1.73 is that of beam (1).]

No comparable aluminium alloy sections were available for a similar test.

In all fairness, attention should be directed to the fact that only two out of an infinite number of load combinations were tested in these experiments, and only a few extrusions and a small number of span-to-depth ratios were used. It is obvious that to do justice to this subject would require much more time than has been available, much better testing facilities, and a more ready access to all commercially available forms of these extrusions. As mentioned earlier, such research could, and should, be undertaken by the manufacturing concerns, and they should present in table form information similar to that contained in the standard steel handbooks, information which gives, for example, such vital data as which of the two criteria controls in the design and in what circumstances—the deflections or the average shearing stresses in the webs of thin sections.

EVALUATION OF THE THEORY OF LIMIT DESIGN—CONCLUSIONS AND RECOMMENDATIONS.

The methods of Applied Elasticity make use of the elastic limit stress as the one and final criterion of strength. After all the necessary equations have been solved and all the arithmetical gymnastics performed, there still remains that one final stipulation: at no one point in a structure, regardless of whether that structure is simple or redundant, must the elastic limit stress be exceeded; and, to make certain that this shall be achieved, a factor is applied to the elastic limit stress to obtain the so-called working stress. This method is unsatisfactory for three important reasons.

First, this factor, often called a safety factor, is in reality a factor of ignorance.⁽¹⁷⁾ It is intended to take care of the many unaccountable factors that enter into the actual construction and service after the plans leave the designer. The factor as such should be applied to the maximum loads to obtain working loads.

Second, the method conveniently ignores secondary stresses which are difficult to compute, as well as stresses around rivet holes in the connections, around points of application of loads and reactions, around junction points of web and flange in I-sections. Thus there is involved some sort of a double standard. There are cases in which the elastic limit stress must control, and other cases in which, frankly, it does not have to control. Even if these secondary items are ignored and only pure beam action is considered, the elastic limit stress is still an inadequate criterion of the strength of redundant structures, especially those built of the light metals, as has already been demonstrated. Because all modern construction is based on continuity, some other criterion should be adopted. The maximum elastic deformations of strictly comparable non-redundant systems is proposed as a suitable yardstick for allowable deformations of redundant systems.

Third, the assumptions made in the use of the Applied Elasticity equations are seldom satisfied in practice. The alignment of points of support is never perfect. Members which are assumed to be perfectly straight rarely satisfy this condition. They are usually sprung into place. If the middle support of a beam resting on three supports were just a tiny fraction of the span higher than the other two supports, the elastic limit stress at the centre would be reached long before the loads would attain even the values based on elastic behaviour. Such misalignments cannot be controlled from the designer's desk. The best that can be done is to impose certain tolerances in shop or field construction. This means that the significance of elaborate arithmetical computations is all but lost, and that the time spent on them is all but wasted. If only approximate values can be obtained, it would be better and more logical in the circumstances to apply Kist's principle to design.

As has been stated earlier, the fact has to be faced that the very nature of the stress-strain characteristics of the light metals precludes any exact determination of limiting values. Therefore, whatever method is used when the elastic limit is passed can only be an approximate method. Applied to structures made of these metals, Limit Design takes its place among other approximate methods. The only virtue it may claim is its underlying philosophy of according primary importance to primary considerations and secondary importance to secondary considerations. While Applied Elasticity is concerned with, among other things, the determination of the instantaneous stress distribution, Limit Design ignores it completely. Once the limiting conditions have been decided upon and the structure has been designed on the basis of these conditions, then whatever is the stress distribution for loads below the limit loads is of no consequence. It is only of academic interest. Its determination does not contribute to the theory of strength. In certain cases it might

be of interest to know how the various elements behave in all the stages leading to the limiting conditions, but such knowledge would be of more value to analysis than to design.

Admittedly, both Applied Elasticity and Limit Design are, in the words of Professor Kist, "only caricatures of reality,"⁽⁶⁾ but the basic concept of Limit Design is more faithful to reality. How close its assumptions can be realised in the over-loaded limit conditions depends on the ductility of the material of which the component parts of the structure are made and also, upon the efficiency of the connections.

Ductility is a property of metals for which, unfortunately, neither industry nor the profession has as yet devised an absolute measure. Yet ductility is at least as important as strength for structural purposes. Professor George Welter of the Ecole Polytechnique, in his as yet unpublished discussion of Professor Van den Broek's paper, "Evaluation of Airplane Metals,"⁽¹⁸⁾ hit the nail on the head when he said, "Only two indirect indices, the elongation and the reduction in area, are nearly all we possess on which to estimate this basic characteristic. We do not even know the exact meaning of these values in relation to ductility." He also says, "Non-symmetrical loading, insufficient alignment of the elements, are in general the external sources of uncontrollable, localised working stresses. Without any doubt, ductility is the most effective insurance against high stress concentration factors."

To illustrate what Professor Welter means, attention is invited to the data contained in Figs. 2 and 6, the stress-strain curves of Dowmetal magnesium alloy ZK60 and of cold rolled steel, respectively. The ZK60 specimen manifested a 28 per cent. elongation in two inches and a 48 per cent. reduction in area. The cold rolled steel specimen elongated only 18 per cent. and reduced its area by the slightly greater amount of 57 per cent. And yet when $\frac{5}{8}$ in. square bars of the two alloys were bent in the bending machine

over the same radius of $\frac{1}{4}$ in., the cold rolled steel bar showed separation cracks only at, or near, 180 degrees, while the ZK60 bar fractured along a 45-degree line at approximately 45 degrees of bend. (See Plate II.) It is recommended that the bend test be standardised as a measure of ductility.

Until absolute standards and measures are set for this property of metals, ductility, the limitations of Limit Design can be described only in the most general terms. If, as seems to be the case, the bend test is a better index of ductility than elongation and reduction in area, then much caution should be exercised in applying the simplified methods of Limit Design to the two alloys of Dowmetal that were tested, particularly 01HTA. The new experimental alloy, ZK60, does not shatter the way 01HTA does, and it has brilliant

As for the assumption of equalised moments, it is obvious that the moments should not always be assumed equalised. In the two load combinations used in the tests, the ratios in the elastic range of the two highest bending moments were only 1.2 and 1.5 (Figs. (g) and (e)). Suppose such a ratio were 3 or 5 or even higher, no definite answer can be given as to what might occur. This must await a more perfect definition of ductility. It can only be said that the more ductile the material the higher is this ratio for which unity may be assumed in the limit. In extreme cases an idea of how large or how small this ratio should be assumed, could probably be gleaned from the assumed elastic behaviour of the beam. This ratio will be close to unity, or much greater than unity, depending on whether the material has much or little ductility. For extremely brittle materials there is no question but that the elastic limit stress should control in the design, that is control in the strictest sense. For a very ductile material like mild steel there is no necessity of referring to the elastic behaviour. A test, which has not been reported here, was run on a steel bar as a continuous beam on four supports in which the ratio of the two highest bending moments was about 3.7. The load based on equalised moments was not only completely realised, but the beam carried an even higher load.

Much has been written on the subject of what happens and how it happens at the one critical section in a simple beam when the fibres are stressed beyond the proportional limit. Nadai,⁽¹⁹⁾ Timoshenko,⁽¹⁴⁾ and more recently Scott,⁽²⁰⁾ Cozzone,⁽¹⁵⁾ Rappleyea and Eastman,⁽²¹⁾ and Peterson,⁽²²⁾ to mention but a few, have contributed to this subject. In the final application to the design of structures, however, this study, like the study of the stresses at the junction of the web and the flange in an idealised I-section, is just an interesting detail. The really important thing to know is *what* a section contributes, *when* in the course of loading is it contributed, and *how long* is it contributed in the

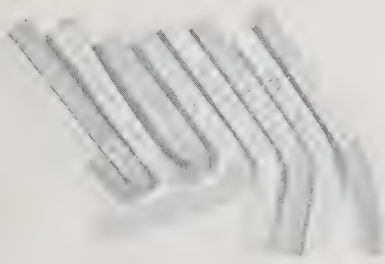


Plate II.

$\frac{5}{8}$ in. square bars bent in the bending machine over a radius of $\frac{1}{4}$ in. Left to right: Hot rolled steel, cold rolled steel, aluminium alloy 61ST, aluminium alloy R303-T315, aluminium alloy 24ST, magnesium alloy ZK60, magnesium alloy 01HTA.

possibilities, its strength being substantially the same as that of 01HTA. 01HTA shatters much like shrapnel, making testing rather risky. Without doubt much of this material has to be scrapped when extrusions of it are sprung into place in shop construction. No difficulties in testing were encountered with the aluminium alloys. Typical failures of these different alloys are shown in Plates III and IV.

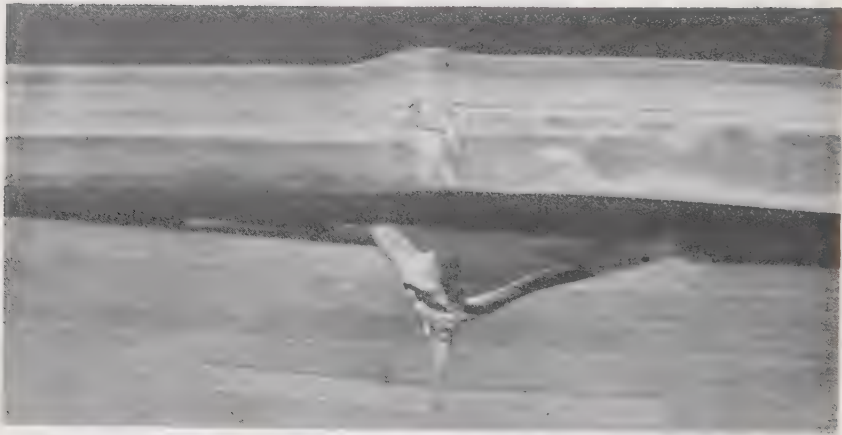


Plate III.
Typical local failures of magnesium alloy 01HTA I-extrusions.

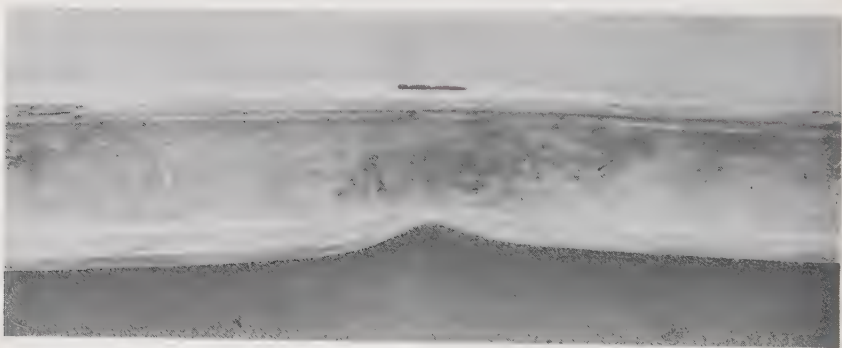


Plate IV.
Typical local failure of aluminium alloy 61ST I-extrusions.

functioning of a redundant structure of which it is an integral part. The most sensitive feeling for the physical is essential in order to appreciate fully the interaction between the redundants and the "primary" system in a highly complex structure.

In conclusion then it may be said that:

(1) The simplified methods of Limit Design, developed for specific application to

steel structures, can be applied with safety to structures made of the alloys of magnesium and of aluminium, provided that some suitable value of stress be used in lieu of the yield point stress in steel.

(2) The peculiar shape of the stress-strain curves of the light metals actually constitutes an advantage in that relatively higher loads can be carried by redundant structures of the

light metals (as compared to similar non-redundant systems) than can be carried by redundant steel structures (as compared to similar non-redundant systems).

(3) The comparative brittleness of the light metals, however, constitutes a distinct disadvantage, which cannot be clearly reduced to exact figures until an answer is found to the question: What is the minimum of ductility consistent with safety, considering the differences in the requirements of various constructions?

(4) Thought should be given to adopting a criterion of strength more logical than the elastic limit stress. For the present there does not seem to be a more logical yardstick than the elastic deflections of non-redundant systems.

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